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# Vernier Scan Analysis for PHENIX Run 15 p+p Collisions at $\sqrt{s}$ = 200 GeV

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# *Abstract*

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**Vernier Scan Analysis for PHENIX Run 15 p+p Collisions at $\sqrt{s}$ = 200 GeV**

by Gregory J. OTTINO

This analysis focuses on the computation of the minimum bias detector cross section via a Vernier Scan analysis for p+p collisions at center of mass energy, $\sqrt{s} = 200$ GeV, at the Pioneering High Energy Nuclear Interaction eXperiment (PHENIX) during Relativistic Heavy Ion Collider (RHIC) Run 15. Of fundamental interest in particle physics is quantifying the probability that a certain final state will result from a set of initial conditions. In order to do this, a Vernier Scan is performed to characterize the intensity per unit area of the collisions, known in particle physics as the luminosity; this procedure forms the background of cross section calculations. This analysis discusses both the computation of, as well as corrections to the Vernier Scan measurement. In addition to the first order calculation of the luminosity, several important corrections to the measurement, including accounting for a nonzero crossing angle of the beam and a correlation between the width of the beam in the x-y plane and the position of the beam in the longitudinal dimension, are applied to the calculation. The final quantity obtained is the minimum bias trigger cross section, $\sigma_{BBC}$, which can be converted into the luminosity for any particular data set from Run 15. The average $\sigma_{BBC}$ for Run 15 $p$+$p$ was found to be $\sigma_{BBC} = 30.0 \pm 1.8(stat) \pm 3.4(sys)$ mb.

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# Chapter 1

# Theory & Experiment of Cross Sections & Luminosity

## 1.1 The Physics of Cross Sections and Luminosity

### 1.1.1 Luminosity

Luminosity in particle physics is the proportionality between the rate of a particular process, X, and that process's cross section, $\sigma_X$ [1].

$$\frac{dX}{dt} = \mathcal{L}\ \sigma_X$$

where $\mathcal{L}$ is the luminosity, measured in $cm^{-2}\ \ s^{-1}$ or, since cross sections in particle physics are so small, using millibarns ($10^{-24}cm^2 = 1mb$). The luminosity can be thought of as characterizing the experimental conditions in a collider. Closely related to the luminosity is the integrated luminosity, L, which is a measure of the number of times a process X will occur ($N_X$) during the integration time period,

$$N_X = L\ \sigma_X$$

By checking a dataset for the number of events X, it is therefore possible to extract the cross section of any process X that occurs. In particle physics experiments, the amount of data is often given in terms of integrated luminosity.

In a particle collider, bunches of charge particles are accelerated to high energies and collided to create interactions. The beams that are collided consist of discrete bunches passing through each other, causing one or more collisions for each bunch crossing. The bunches are modeled as a density of particles moving through space. Locally near the point where the bunches interact, one bunch is considered to be moving to the left in z, denoted with a + sign, and the other bunch is considered to be moving to the right in z, denoted with a - sign. The bunches have a time independent 3D structure, where the time dependence translates the bunches in their respective directions in the z dimension. A first order expression can be derived for the luminosity by assuming the bunch shapes are uncorrelated in all dimensions and the collisions are head on. The luminosity is treated as the overlap integral of two arbitrary density functions, representing the colliding bunches, that depend on x,y,z, and t. The second important assumption about the structure of the bunches is to assume the density is a product of a Gaussian in all dimensions. This implies, that for the $\pm$ bunch, the density can be denoted as $\rho_{\pm}$ and depends on the number of protons in the bunch $N_{\pm}$, and the Gaussian width in each dimension $\sigma'_{x,\,y\,z}$ where

$$\rho_{\pm} = \frac{N_{\pm}}{\sqrt{(2\pi)^3}\sigma'_z\sigma'_x\sigma'_y} e^{(\frac{-x^2}{2\sigma'_x} - \frac{y^2}{2\sigma'_y} - \frac{(z\pm ct)^2}{2\sigma'_z})}$$

The general overlap integral depends only on the densities of the bunches, the bunch crossing frequency $f_0$, and the number of bunches in the beam $N_b$. It is given by,

$$\mathcal{L} = 2f_0 N_b \iiiint_{-\infty}^{\infty} \rho_+(x,y,z,ct)\rho_-(x,y,z,ct)\, dx\, dy\, dz\, cdt$$

where the factor of two is a result of the assumption of head on relativistic collisions.

Using the $\rho_{\pm}$ with a 3D Gaussian and integrating over each dimension, a final, first order formula for the luminosity is obtained,

$$\mathcal{L} = \frac{N_+ N_- f_0 N_b}{4\pi\sigma'_x\sigma'_y}$$

## 1.2 The Vernier Scan

Given the important role luminosity plays in determining cross sections, it is important to characterize it experimentally. This is often done at colliders using a procedure known as the Vernier Scan. Originally proposed in 1968 by physicist Simone Van der Meer, the Vernier Scan has long been the standard for measuring luminosity at RHIC, as well as the Large Hadron Collider and other, earlier accelerators. The total luminosity available to an experiment is determined by the minimum bias trigger, which is effectively the condition to begin recording data into the Data Acquisition System (DAQ). For a given experiment, a minimum bias detector is often used as a luminosity counter. The minimum bias trigger (MB) has a cross section, $\sigma_{MB}$, the fraction of the total cross section as seen by the minimum bias trigger. For any physics data set, integrated luminosity is found by the relation,

$$L = \frac{N_{MB}}{\sigma_{MB}}$$

where $\sigma_{MB}$ is the quantity determined by the Vernier Scan analysis, and it comes from another, similar expression using the instantaneous luminosity and collision rate,

$$\sigma_{MB} = \frac{R_{MB}}{\mathcal{L}_{\mathcal{MB}}}$$

Here, $\mathcal{L}_{MB}$ is the luminosity detected by the minimum bias trigger, and $R_{MB}$ is the collision rate seen by the same MB trigger. $\mathcal{L}_{MB}$ is equal to $\mathcal{L}_{delivered} * \epsilon_{MB}$ where the former is the delivered luminosity and the latter is the minimum bias trigger efficiency.

The collision rate is measured by looking at the ratio of the number of minimum bias triggers over time. The delivered luminosity is calculated from the Vernier Scan analysis:

$$\mathcal{L}_{delivered} = f_0 \frac{N_+ N_-}{4\pi\sigma_{x'}\sigma_{y'}}$$

as derived above. The parameters $\sigma_{x'}$ and $\sigma_{y'}$ are determined via the Vernier Scan.

The Vernier scan procedure consists in scanning one beam across the other in discreet steps, measuring the collision rate at each step position. This enables the measurement of the width of the convolution of the two bunches, in the x-y plane, by looking at the distribution in rate vs. position space. The number of protons is measured directly by using an induction detector that measures the charge in the bunch and converting that to a number of protons.

Given the nature of the Vernier Scan, the direct values of $\sigma_{x'}$ and $\sigma_{y'}$ are not found, and instead the overlap width in each dimension is found. Since the widths are considered to be Gaussian and equivalent for both bunches, the overlap of the beams is considered to be an overlap as a function of time of two Gaussians with a simple relation between their widths, $\sigma_{x(y)} = \sqrt{2} * \sigma_{x(y)'}$. This leaves a final luminosity formula in terms of known or observable parameters:

$$\mathcal{L}_{delivered} = f_0 \frac{N_+ N_-}{2\pi\sigma_x\sigma_y}$$

## 1.3 The PHENIX Experiment

The Pioneering High Energy Nuclear Interaction eXperiment (PHENIX) is a general purpose detector located at the Relativistic Heavy Ion Collider (RHIC) on the campus of Brookhaven National Labs (BNL) [2]. PHENIX was commissioned to study both hot nuclear matter, the quark gluon plasma, and the proton spin problem. Like all similar collider experiments, a large component of the physics involves investigating the production cross section of various processes that result from the proton proton collisions. The importance of the Vernier Scan analysis is to enable the computation of those cross section by calibrating the minimum bias trigger detector efficiency to give an accurate measurement of the luminosity. RHIC collides both protons as well as heavy ions. This analysis focuses on p+p collisions which occurred at a center of mass energy of $\sqrt{s} = 200$ GeV.

The PHENIX experiment consists of several detector subsystems including time of flight detectors, calorimeters, tracking detectors, and a muon system [2]. A schematic diagram of the detector system is given below in figure 1.1. The main detectors of interest in this analysis are the global counters, the Beam Beam Counter (BBC) and the Zero Degree Calorimeter (ZDC).

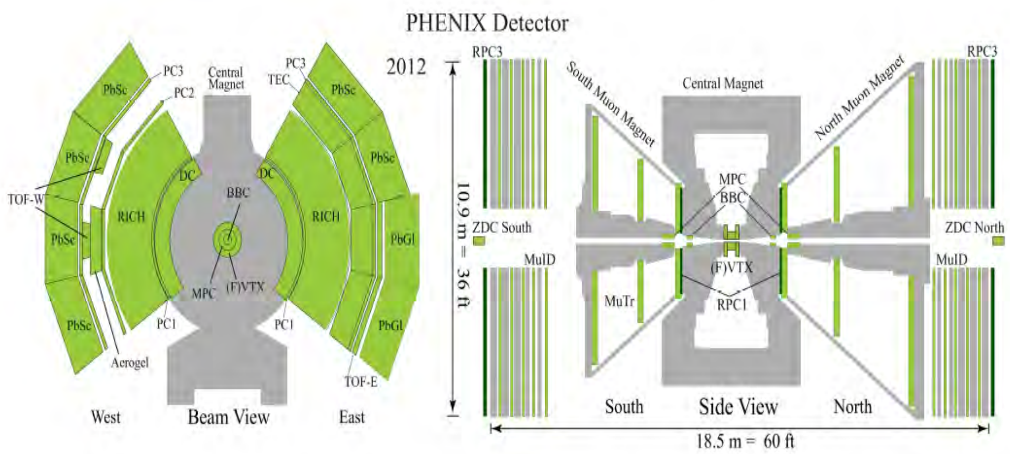



FIGURE 1.1: The PHENIX Detector

The BBC's are a pair of multipurpose detectors, and for the Vernier Scan analysis they act as a minimum bias luminosity counter and vertexing detectors [3]. The

BBC's each consists of 3 cm quartz radiator crystals that are read out by 64 PMT's. The detector units are mounted at $\pm 144$ cm from the nominal experimental interaction point (IP) in the longitudinal direction. They have a coverage in psuedorapidity of $3.0 < |\eta| < 3.9$. The BBCs are able to vertex events using time of flight with a resolution of $52 \pm 4$ ps, which gives a spacial resolution of 1 cm. Since the BBCs are close to the IP, the vertexing precision is dependent on the vertex position and the precision decreases further from the interaction point. Additionally, since the BBC uses time of flight for vertexing, if multiple collisions occur in a single bunch crossing, the BBC is unable to distinguish the collisions, leading to the possibility of under counting the luminosity. This issue is explored in further detail in the analysis section.

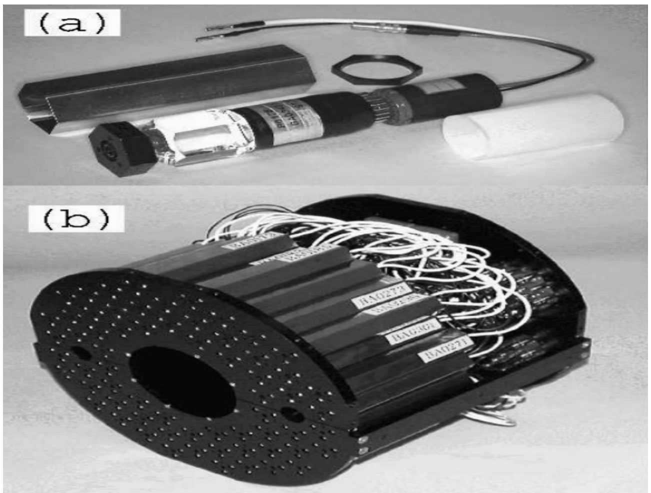


FIGURE 1.2: (a) BBC PMT and Crystal (b) Assembled BBC Module

For the purpose of the Vernier Scan and PHENIX in general, the BBC serves as the minimum bias trigger, and the ultimate parameter of interest is $\sigma_{BBC}$, which corresponds to $\sigma_{MB}$ in the Luminosity equations above. The maximal rate used is the maximum BBC rate. The luminosity available to the experiment, and therefore available to compute cross sections, is only the luminosity seen by the BBC. For this reason the BBC serves as PHENIX's luminosity counter.

The ZDCs are two symmetric hadronic calorimeters located very far from the interaction point at $\pm 18$ m [4]. This enables the ZDCs to detect very low transverse momentum particles, $|\eta| > 6$, which is crucial in determining the centrality in heavy ion collisions. For the Vernier Scan, the ZDCs serve as a z vertex detector using time of flight difference. Due the small geometric acceptance, the ZDCs see very few events, limiting the effect of multiple collisions and enabling corrections to BBC data for effects that depend strongly on the z vertex position. The ZDC's also sees the entire interaction region, allowing for corrections to effects that consider scales in the transverse dimensions that are greater than the BBC's. The ZDC can determine z vertices to a precision of $\pm 15$ cm.

For the Vernier Scan, the ZDCs are used to correct for any longitudinally dependent effects in the analysis. Given that all the formulas derived above depend on the approximation of uncorrelated densities in all dimensions, to correct for this oversight the true shape of the bunch is later applied as a correction. The ZDC is the only detector that can access the entire bunch in the longitudinal (z) dimension, making it critical to make such z dependent corrections.

Several of the parameters of the Vernier Scan analysis are set or measured by the Collider Accelerator Department (CAD) at BNL and incorporated into this analysis.

The number of protons in each bunch in the beam is measured using two separate induction detectors, the Wall Current Monitor (WCM) and the Direct Current Current Transformer (DCCT) [5]. These detectors are both induction detectors, reading out a current as the bunches pass through a closed circuit, generating an induction current. The WCM is a fast detector, giving bunch by bunch readouts of the number or protons. The DCCT is a slower detector, but it has a lower systematic error and is less sensitive to debunching effects. Both detectors are used to generate the final value for the number of protons in each bunch. The bunch crossing frequency, $f_0$, is set by CAD to be about 78 kHz meaning a particular bunch from one beam crosses a particular bunch from another beam 78 thousand times per second.

# Chapter 2

# Vernier Scan Analysis

The Vernier Scan analysis for PHENIX consists of several distinct experimental techniques over several data sets, outlined in the sections below.

## 2.1 Analysis Data Sets

The Vernier Scan analysis consists of data taken at the end of a 90 minute period of data collection, known as a run at PHENIX. In order to establish a statistically significant measurement, several data sets are generated over the entire course of p+p running. The Vernier scans occur for a period of about 15 minutes, with 24 steps total. The scan begins with the beams in a maximally overlapped configuration, as they would be during normal running, then moving outward in steps of between 100 and 250 $\mu$m. First the horizontal direction was scanned, then the vertical. A summary table of the scans taken over all of 2015, known as Run 15, is in table 2.1.

| Run | Fill | Comments |
|---|---|---|
| 424347 | 18721 | Run Control Server Failure |
| 426254 | 18776 | - |
| 431624 | 18942 | Test of Diagonal Scan (Omitted in Analysis) |
| 431723 | 18943 | Diagonal and Horizontal/Vertical Scans |
| 431942 | 18952 | CLOCK Trigger Enabled |

TABLE 2.1: Run 15 Vernier Scan Summary

The Run Control Server failure was a communication issue between the PHENIX control room and the Collider Accelerator control room. This issue did not ultimately affect the data collected for run 424347. The diagonal test scan was a proof of concept, in order to more accurately confirm the assumption of Gaussian distributions in the x-y plane. However, data collection issues resulting from an incorrect trigger configuration prevented the diagonal scan from being utilized. The final scan was taken after the trigger configuration issues were resolved (i.e. the CLOCK trigger was enabled).

### 2.1.1 Trigger Configurations Issues

An unfortunate issue with PHENIX p+p Vernier Scan for Run 15 was the correct trigger configuration was not enabled except for the last scan. Normally for the Vernier Scan, two types of data are used to compute the rates at each displaced step position, the BBC_MB scalar counts and the CLOCK scalar counts. The former records the number of minimum bias trigger over the course of a run, and the latter is a scaled value of the time. The rates are then calculated by taking the ratio of the number of BBC_MB scalars and CLOCK scalars, then scaling the CLOCK to recover time. However, for the majority of Vernier Scan data sets in Run 15, excluding 431962, the CLOCK trigger was not enabled in the DAQ. Instead of simply dividing two trigger counts, a raw number of cumulative triggers and an associated EPOCH time stamp was used to compute rates by taking the difference in counts between two sequential steps and dividing by the difference in epoch time. Unfortunately, the rates extracted via this method proved to be unreliable, making it impossible to perform the full Vernier scan procedure on the majority of the data sets. However, certain aspects of the scan, in particular the values of the BBC trigger efficiency, were extracted from each run, since it did not require the workaround method in the same way the rates did.

The main issue with the workaround method was that the data needed to be corrected for livetime effects. The livetime of the DAQ is the fraction of time that the DAQ is taking data. When an event arrives at the DAQ, no more data is analyzed until the current level 0 analysis is completed. Therefore, while the DAQ is busy, some data is not accepted. However, scalar counters still record the number of total triggers, and these scalars are not overwhelmed by the data taking rate. Therefore, the livetime is the number of scalars accumulated when the DAQ is live divided by the total number of scalars. The livetime changes dramatically and nonlinearly between Vernier Scan steps, because the data collection rate can greatly vary given how much of the beams are overlapping. Since the beam widths are Gaussian, a linear change in beam overlap can cause a dramatic drop in the rate. As the rate falls, the livetime rises because the DAQ is busy more often.

The correction that was attempted depended on data packets that occurred approximately every 30 s. Since the Vernier scan step duration lasted between 30s and 1 minute, there was the possibility that two data packets would be contained within a single steps, while other steps had only one or zero packets. Since the difference between two data packets was needed to compute the livetime for a step, only those steps which had two data packets could be utilized. Enough steps had two data packets such that a beam profile was able to be extracted from each run. Unfortunately, the data was still unusable due to issues with the rate calculation, as is explained in the subsection on the livetime correction in the next section.

## 2.2 Event Rate and Step Position

The data that is generated specifically for the Vernier Scan relates to the transverse beam profiles, $\sigma_{x(y)}$. The physical scan that occurs is the process of sweeping one beam across the other at discrete steps in the transverse (xy) plane, generating a data point in Rate vs. Position space. The rate data for a single transverse dimension are plotted first separately in one dimension, then the rate vs. position is plotted for the entire transverse plane in a two dimensional plot. The 1D plots are fit with a simple Gaussian function, with the normalization equating to $R_{max}$ and the width corresponding to $\sigma_{x(y)}$. The 2D case is similarly fit with the product of a Gaussian function in x and in y with the fit parameters corresponding to the same quantities as the 1D case. While the fit parameter values for the 1 and 2 dimensional fits differ only by $1\%$, the 2D fit is preferred given that the beam may not return to exact maximal overlap in later steps during the scan.

The step position has, in previous years, been taken from the CAD's Beam Position Monitors which record the beam location in the x-y plane. However, given known and possible issues in the BPM's measurements that are difficult to calibrate, the nominal step positions set by CAD were used instead. Since the step position was not a measured quantity, it had no corrections. See Figure 2.1 and 2.2 for examples of plots of the BPM readings vs. time.

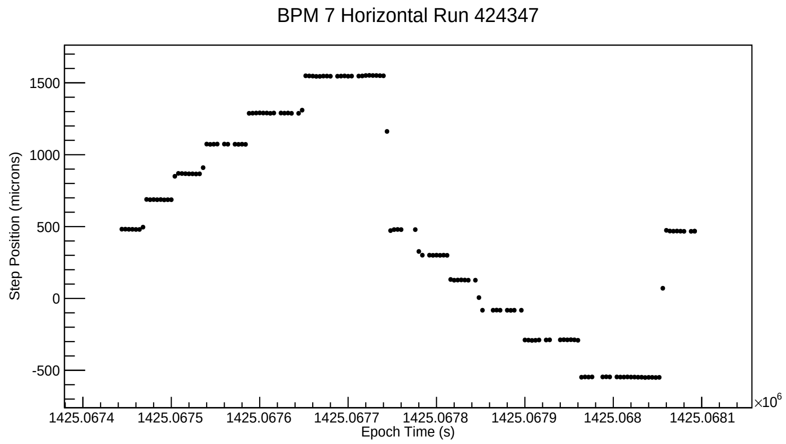



FIGURE 2.1: BPM Run 424347 Horizontal. This is an example of the BPM reading as the Vernier Scan moves through its steps. The step sizes are approximately 150$\mu$m.

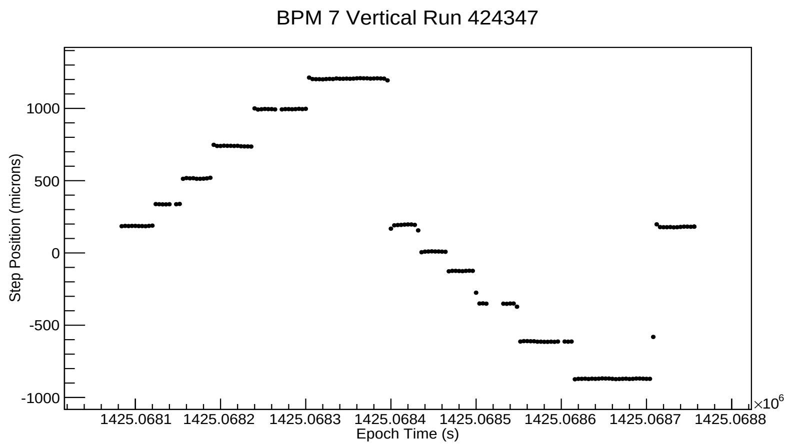



FIGURE 2.2: BPM Run 424347 Vertical. This is an example of the BPM reading as the Vernier Scan moves through its steps. The step sizes are approximately 150$\mu$m.

The rate data was extracted by dividing the number of BBC triggers by CLOCK triggers or by differences in epoch time. The rates were averaged over each step, giving a corresponding rate to each step position. The plots below show a comparison of run 431962 for both the traditional rate calculations and the work around method. (Note:These plots are fully corrected; a discussion of the corrections is in subsequent sections.)

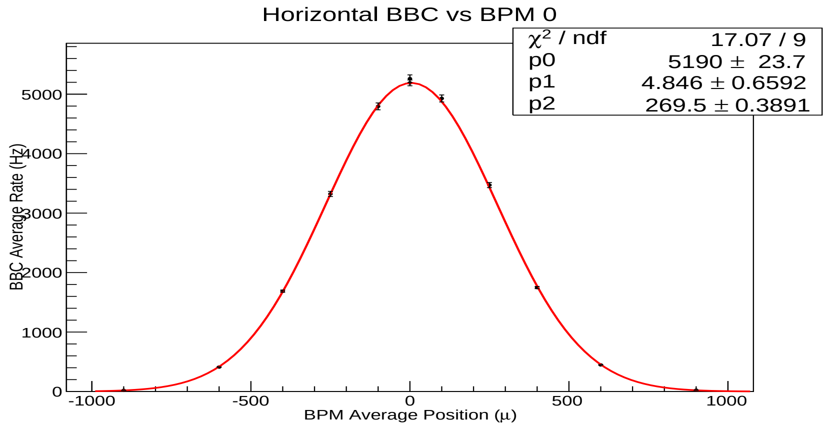



FIGURE 2.3: Run 431962 Horizontal Profile CLOCK Scalar. The Gaussian fits to Rate vs. Step plots are shown here, along with the fit parameters. p0 corresponds to the maximum rate, and p2 is the Gaussian width of the overlap of the bunches.

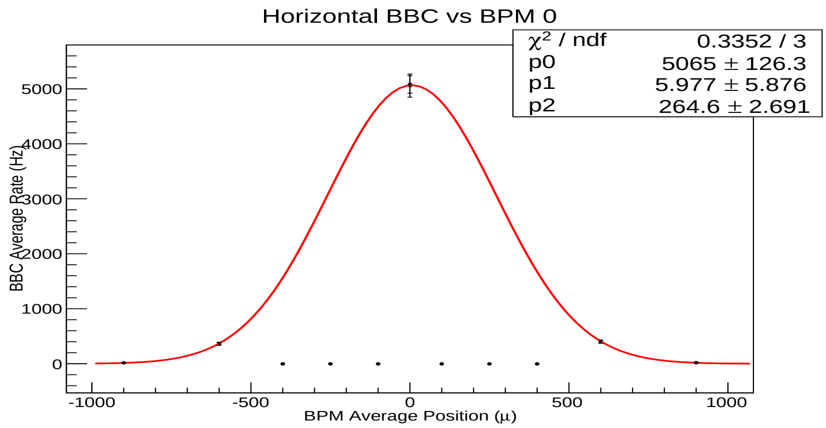



FIGURE 2.4: Run 431962 Horizontal Profile workaround method. The Gaussian fits to Rate vs. Step plots are shown here, along with the fit parameters. p0 corresponds to the maximum rate, and p2 is the Gaussian width of the overlap of the bunches. The zero points that are not accounted for are those for which livetime corrections are not available, and they do not inform the fit.

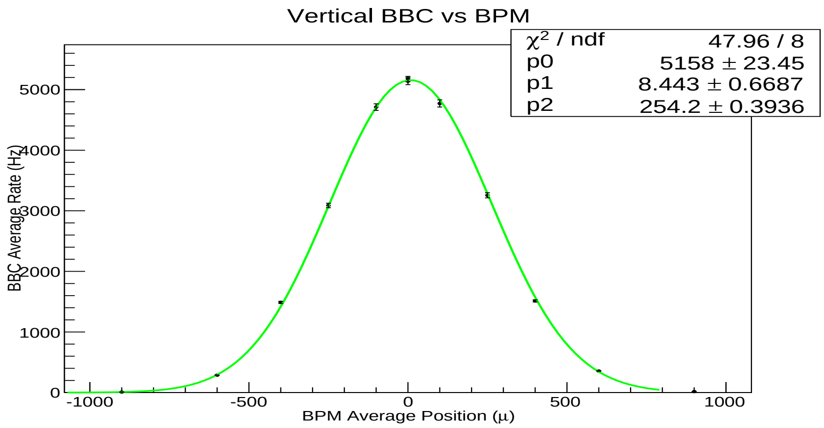



FIGURE 2.5: Run 431962 Vertical Profile CLOCK Scalar. More Gaussian fits where p0 corresponds to the maximum rate, and p2 is the Gaussian width of the overlap of the bunches.

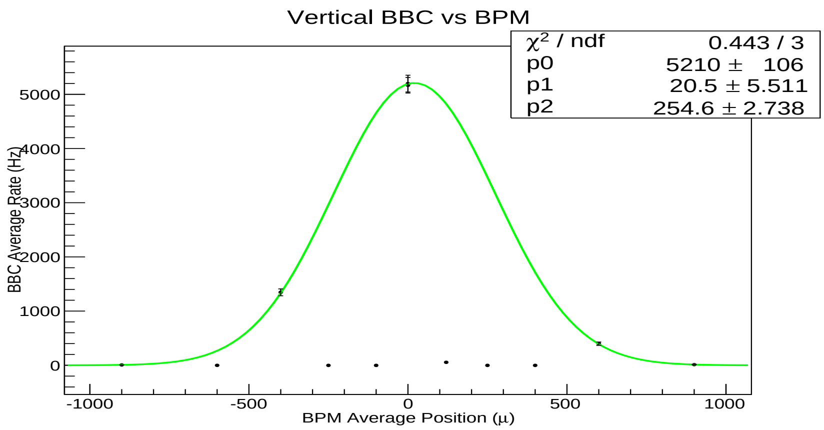



FIGURE 2.6: Run 431962 Vertical Profile workaround. More Gaussian fits where p0 corresponds to the maximum rate, and p2 is the Gaussian width of the overlap of the bunches. The zero points that are not accounted for are those for which livetime corrections are not available, and they do not inform the fit.

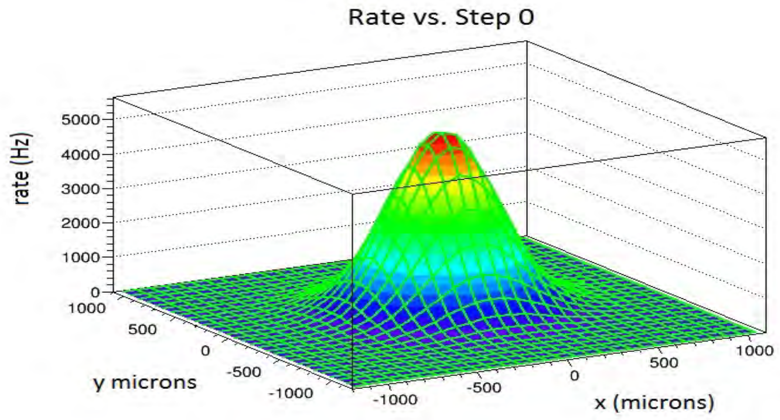



FIGURE 2.7: Run 431962 2 Dimensional Profile CLOCK Scalar. This is an example of the three dimensional fits which are preferred for extracting the maximum rate. The fit parameters are not shown here.

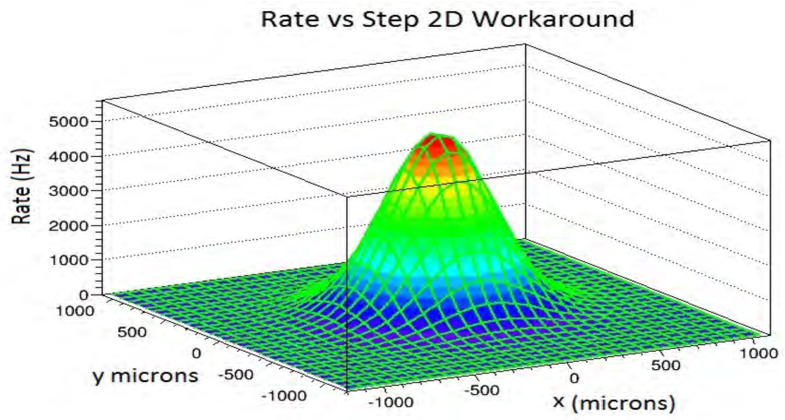



FIGURE 2.8: Run 431962 2 Dimensional Profile workaround. This is an example of the three dimensional fits which are preferred for extracting the maximum rate. The fit parameters are not shown here.

### 2.2.1 Failure of the Workaround Method

An unfortunate consequence of the incorrect scalars being enabled at the time data was taken was that the rates were not able to be corrected for livetime on a bunch by bunch basis. A naive assumption would imply that each crossing of bunches would have similar numbers of protons and densities which would lead to similar rates. Upon investigating systematic effects, however, the livetime correction introduced systematic effects into the data that varied with the bunch crossing. In particular, the rate with the livetime correction tended to fall as the number of protons increases. Given that the bunch widths are constant (with only statistical error) the rates should increase as the number of particles increases.

During the run for which the CLOCK scalar was available, this was the case, and since the final value of $\sigma_{BBC}$ has the relationship,

$$\sigma_{BBC} \propto \frac{R}{N_b N_y}$$

the increase in the number of protons leads to an increase in the rates, which balances out to give an approximately constant value for $\sigma_{BBC}$ (see Results and Conclusion). See figure 2.9.

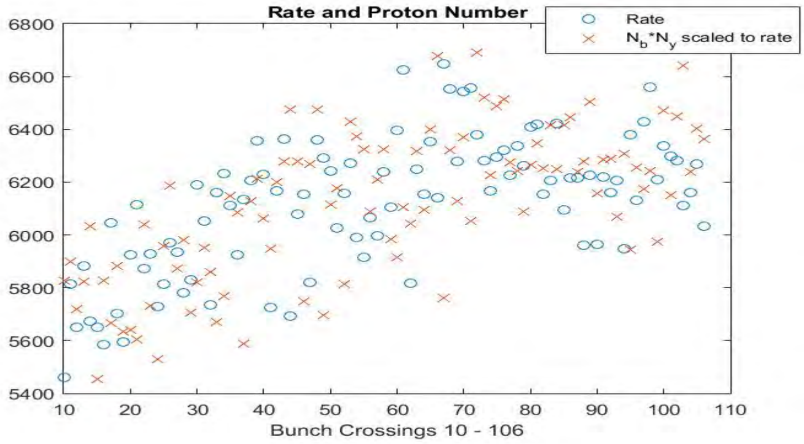



FIGURE 2.9: Rate and $N_y N_y$ as a function of bunch crossing number for run 431962. As expected, as one the number of protons goes up, so does the rate

For comparison, figure 2.10 shows the behavior of run 426254. As the number of protons increases, the rate tends to decrease. Although physically this is incorrect behavior, the issue is present and lies within the PHENIX DAQ. Since the live time of the DAQ is extremely sensitive and responds nonlinearly to changes in the rate, even the variations that occur between the bunches in one Vernier scan make it impossible to use a bunch averaged livetime correction. Specifically, as the rate increases, the livetime of the DAQ falls. If the true, unknown livetime of a particular bunch crossing is smaller than the average value of the livetime, the correction to that particular bunch crossing will calculate the rate to be smaller than it is. This causes the measured rate to fall as the true rate increases. Because the effect is nonlinear and the relationship is not understood, it cannot be corrected for without introducing a large amount of systematic uncertainty. Since the bunch averaged livetime is the only data available, the runs 426254 and 424347 were not used in order to avoid introducing poorly understood systematic effects into the data. When the rate data was not used, certain parameters such as the BBC efficiency were still computed in order to check the consistency of the runs. However, the final calculated value of $\sigma_{BBC}$ is a result of only run 431962.

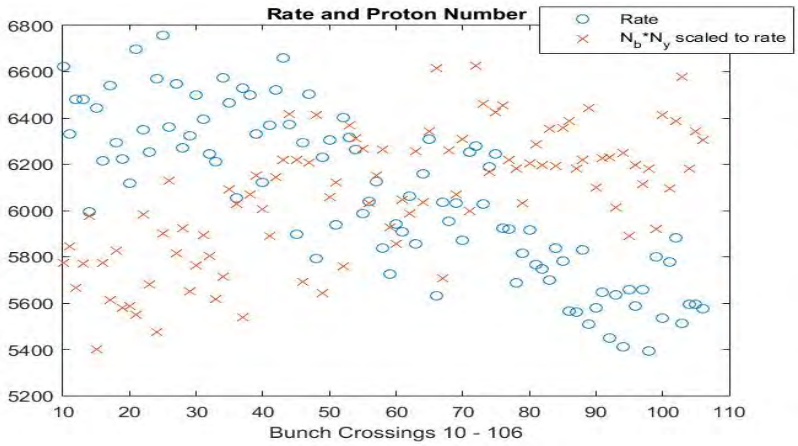



FIGURE 2.10: The rate and scaled number of protons for the workaround on run 431962. The decline in the rate, even as the number of protons increase, implies that the livetime correction cannot be applied as a bunch average correction, but instead must be considered on a bunch by bunch basis. Since this bunch by bunch livetime information is unavailable, the true rates from the workaround method cannot be calculated.

## 2.3 Proton Density

The total intensity of the beam is the number of protons in one beam, i.e. all the bunches of protons traveling one direction, multiplied by the number of protons in the other beam (called arbitrarily 'blue' and 'yellow' in PHENIX). This is broken down for each bunch crossing to find a density for each bunch in both beams. The parameter comes directly from CAD measurements generated by the Wall Current Monitor (WCM). The wall current monitor takes readings for each bunch in time bins averaged over one second. The values for $N_b$ and $N_y$ are taken from a period of approximately 100 seconds prior to the beginning of the Vernier Scan, averaged over time and used in the final computation of $\sigma_{BBC}$. However, the WCM measurements are not particularly accurate and the detector has a relatively high amount of systematic uncertainty ( 2%) as it is sensitive to debunching effects. Therefore, each WCM reading is normalized with respect to another proton counter, the Direct Current Current Transformer (DCCT). Since the DCCT acts over a longer timescale, it is less sensitive to debunching than the WCM, and it has a smaller degree of systematic uncertainty ( 0.2%). The normalization consists of averaging the WCM and DCCT readings over the same 100 second prior to the scan (t) and over all bunches for the WCM (bunches). Then, for the $i^{th}$ bunch:

$$N_{i,b(y)} = \frac{\sum\limits_{t} DCCT}{\sum\limits_{t} \sum\limits_{bunches} WCM} WCM_{i,b(y)}$$

This provides an accurate accounting of the number of protons in each bunch.

## 2.4 BBC Trigger Efficiency

The luminosity sampled by the minimum bias trigger is needed for the calculation of the BBC cross section.

$$\mathcal{L}_{BBC} = \mathcal{L}_{delivered} \times \epsilon_{BBC}^{MB}$$

where $\epsilon_{BBC}^{MB}$ is the fraction of the vertex distribution triggered on by the minimum bias trigger of the BBC. Different BBC triggers serve different functions, and the minimum bias trigger has a range of approximately 30 cm. The BBC_wide trigger has a larger range of coverage in z, extending from $\pm 144$ cm. The efficiency of the minimum bias trigger is measured by dividing the number of triggers that occur inside the BBC minimum bias trigger's range in coincidence with the BBC wide trigger, which covers a wider region, by the total number of wide triggers for the BBC according to the equation,

$$\epsilon_{BBC}^{MB} = \frac{N_{BBC}^{MB+wide}}{N_{BBC}^{wide}}.$$

The BBC trigger efficiency is dependent of the position of the collision along the beam axis, in the z-direction. The z-vertex dependence of the efficiency will result in a narrowing of the naive BBC vertex position distribution taken from the data. To correct for this effect, the ZDC is used as the it is located 18 m away from the center of PHENIX and therefore is reasonably free of any z-dependence. The BBC vertex distribution are widened by taking the ratio of two ZDC z vertex distributions. The first is the ZDC z vertex distribution which is the coincidence of the ZDC wide trigger, which covers the entire $\pm 300$ cm of the interaction region, and the BBC wide trigger, which covers the full region of the BBC range of $\pm 144$ cm. The second distribution is the ZDC z vertex distribution with only the ZDC wide trigger. The coincidence histogram divided by the ZDC wide only histogram gives a centimeter by centimeter correction which is applied to all the BBC histograms in the first calculation of $\epsilon_{BBC}$. The final tabulated values for all the runs is in the table below.

| Run | Uncorrected $\epsilon$ | Corrected $\epsilon$ |
|---|---|---|
| 424347 | .381 | .340 |
| 426254 | .383 | .343 |
| 431723 | .386 | .347 |
| 431962 | .390 | .351 |

TABLE 2.2: BBC uncorrected and corrected efficiency results.

## 2.5 Luminosity Fall Off and Multiple Collisions Corrections

In the Vernier Scan analysis, the luminosity is initially calculated to first order operating under several nonphysical assumptions that are corrected in each parameter. There are two main corrections to the event rate at each step, the luminosity fall off correction and the multiple collision correction.

### 2.5.1 Luminosity Fall Off

The initial correction to luminosity accounts for the protons lost over the course of the Vernier Scan. This accounts for protons lost due to collisions and beam scrapping, which causes the rate to fall over time. The WCM values are plotted against time; at each step, the intensity is normalized with respect to the value in the first step. Then, this normalization multiplies the number of protons at the step, correcting for the protons lost over time. Usually, the series of data points are fit to a linear polynomial, however, due to some scans showing more than simple linear fall off, the rate values were corrected individually, on a step by step basis. This correction is small, typically on the order 1-3%. Example plots are included below.

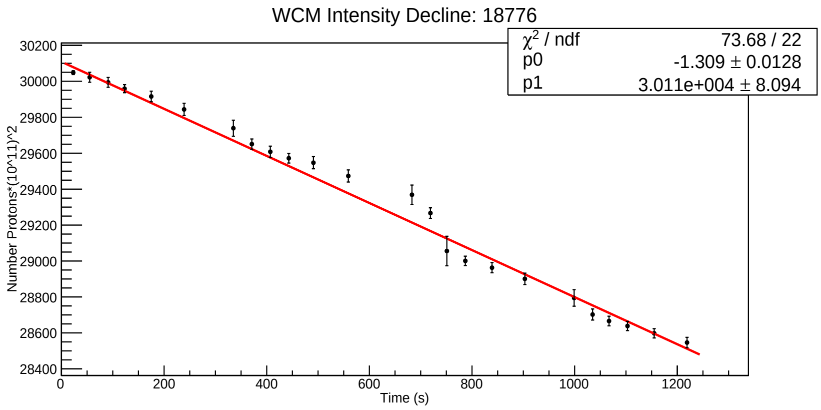



FIGURE 2.11: Luminosity Fall Off Run 18776. This figure shows the change in the product of the number of protons in each bunch as a function of time. This is due to collisions, beam scrapping, and other effects and is important to insure the correct rate is used.

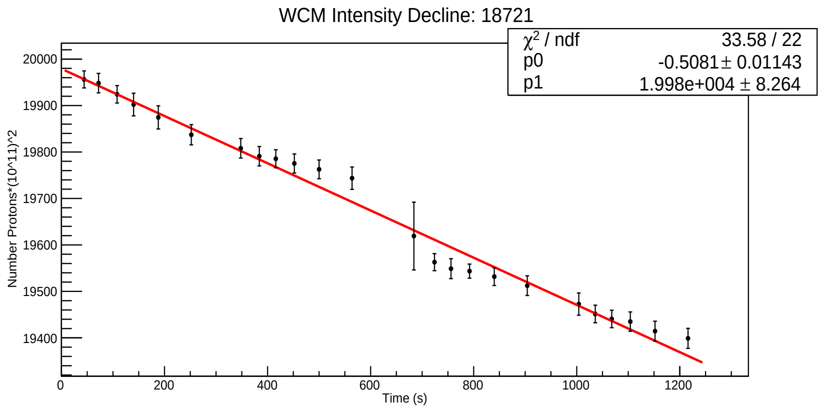



FIGURE 2.12: Luminosity Fall Off Run 18721

### 2.5.2 Multiple Collision Effect

The second necessary correction to the event rate is to account for the multiple collision effect. As a consequence of PHENIX's design as a heavy ion experiment, the BBC is equipped to detect one collision per bunch crossing. In the earlier years of PHENIX, given the lower luminosity of the beams, it was not necessary to account for this effect, as the collision rate per bunch crossing was far less than one per crossing. However, as CAD has increased the luminosity, the effect has become too large to be neglected. The difficulty of analyzing the effect of the multiple collisions lies its opacity to the BBC. Since the minimum bias detector cannot differentiate one collision per crossing from two (or more), a certain amount of uncertainty is introduced. The method used to mitigate this uncertainty was established in the Run 9 Vernier Scan Analysis.

The first important equation describes the true rate [6],

$$R_{true} = \mu * \epsilon_{side}^2 * \epsilon_{BBC} * f$$

where $\mu$ is the average number of collisions in a bunch crossing, $\epsilon_{side}$ is the single side efficiency of the BBC, $\epsilon_{BBC}$ is the trigger efficiency of the BBC discussed previously, and $f$ is the bunch crossing frequency. Since, ideally, the two efficiencies and $f$ are machine parameters, knowing $\mu$ for a bunch crossing would enable the true rate to be calculated. However, $\mu$ cannot be determined directly. Instead, from Poisson statistics the observed rate is expected to be,

$$R_{pred} = \epsilon_{BBC}[1 - 2e^{-2\mu\epsilon_{side}} + e^{-\mu\epsilon_{side}(2-\epsilon_{side})}]$$

Since we understand what the observed rate should be from that formula and $\mu$ is the only unknown parameter, by iteratively minimizing the difference between the actual observed rate $R$ and the predicted observed rate $R_{pred}$ we obtain the correct $\mu$. To do this, Newton's method of root finding is used on the function $f(\mu) = R - R_{pred}$. When the difference between two subsequent iterations is zero, the function is considered to be minimized, and the value of $\mu$ found is used in the true rate equation. A value of $\mu$ is found at each step position, giving a change of between 5-10% at maximal overlap, and <1% at minimal overlap.

It is assumed in this analysis that differences in north and south hit probabilities are small, and a nominal value for $\epsilon_{side} = .79$ is used for this analysis, as established in Run 9. In order to compensate for possible fluctuations, $\epsilon_{side}$ is varied by $\pm 3\%$ which accounts for systematic error in the multiple collisions correction. This error is motivated by the understanding that $\epsilon_{side}$ is not accurately known.

## 2.6 The Hour Glass and Bunch Shape Correction

In the simplest cases, luminosity is calculated as it was described in the first chapter,

$$\mathcal{L}_{delivered} = f_0 \frac{N_b N_y}{2\pi\sigma_x\sigma_y}$$

However, this formula does not account for the possible effects of the z dependence of the bunch structure. The simple formula that is used to approximate luminosity per bunch comes from the parent formula, which integrates over the colliding bunches in x, y, z, and ct:

$$\mathcal{L} = 2f_0 \iiiint_{-\infty}^{\infty} \rho_+(x,y,z,ct)\rho_-(x,y,z,ct)\,dx\,dy\,dz\,cdt$$

Here, $\rho_\pm$ is the structure of each bunch and, assuming Gaussian distributions in x, y, and z$\pm$ct,

$$\rho_\pm(x,y,z\pm ct) = \frac{N^\pm}{(2\pi)^{3/2}\sigma_x\prime(z)\sigma_y\prime(z)\sigma(z)} e^{\left(\frac{-x^2}{2\sigma_x\prime(z)} + \frac{-y^2}{2\sigma_y\prime(z)} + \frac{-(z\pm ct)^2}{2\sigma_z}\right)}$$

The dependence of the x-y plane on the longitudinal dimension is explained physically by the structure of the focusing magnets near the IP. Given that magnet can optimally focus the beam at one point, the IP, the density in x and y depends on how close the transverse plane is to z = 0. To account for the dependence of $\sigma_{x(y)}$

on z, we model the shape as a $\beta$ function [1],

$$\sigma_{x(y)}(z) = \sigma_{x(y)}\prime\sqrt{1+(\frac{z}{\beta^*})^2}$$

In this formula, $\sigma_{x(y)}\prime$ is the actual beam width at the nominal interaction point and $\beta^*$ is the focusing parameter. An example plot from [1]] is shown below. Now, by restating the integral,

$$\mathcal{L} = 2f_0\frac{N_+N_-}{(2\pi)^3\sigma_x\prime^2\sigma_y\prime^2\sigma_z^2}\iiiint_{-\infty}^{\infty} e^{\frac{-x^2}{\sigma_x\prime^2(z)}+\frac{-y^2}{\sigma_y\prime^2(z)}}\,dx\,dy\,\frac{e^{\frac{-(z+ct)}{2\sigma_z}+\frac{-(z-ct)}{2\sigma_z}}}{\{1+(\frac{z}{\beta^*})^2\}^2}\,dz\,dt$$

integrating out the dependence on x and y,

$$\mathcal{L} = 2f_0\frac{N_+N_-}{(2\pi)^3\sigma_x\prime^2\sigma_y\prime^2\sigma_z^2}\iint_{-\infty}^{\infty} \pi\sigma_x\prime\sigma_y\prime\{1+(\frac{z}{\beta^*})^2\}\frac{e^{\frac{-(z+ct)}{2\sigma_z}+\frac{-(z-ct)}{2\sigma_z}}}{\{1+(\frac{z}{\beta^*})^2\}^2}\,dz\,dt$$

and finally simplifying and replacing the beam width with the overlap width measured in the Vernier Scan,

$$\mathcal{L} = f_0\frac{N_+N_-}{2(\pi)^2\sigma_x^2\sigma_y^2\sigma_z^2}\iint_{-\infty}^{\infty} \sigma_x\sigma_y\frac{e^{\frac{-(z+ct)}{2\sigma_z}+\frac{-(z-ct)}{2\sigma_z}}}{1+(\frac{z}{\beta^*})^2}\,dz\,dt$$

we arrive at the integral that shows how the value of $\beta^*$ affects the final luminosity.

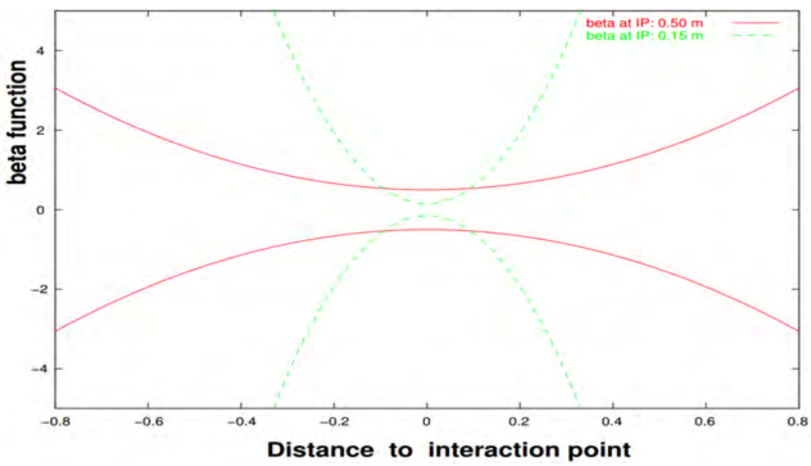



FIGURE 2.13: Example of Different $\beta^*$ [1]

Another crucial correction to the simplistic model is understand the bunches' true longitudinal structure. To first order, the assumption of a Gaussian profile in z is used to understand the luminosity. However, for this correction which is sensitive to slight changes in the z distribution of the protons, it is important to use the true bunch shape, extracted from fine-binned data collected by the WCM. The shape can be well understood by examining the way the bunches are constructed in the RF cavities. The bunches are initially in a single Gaussian bucket which is wider than the final bunch in z. A different RF structure is then used to squeeze the bunches to reduce the z width. However, not all of the protons are captured by this squeezing effect. Therefore, the three Gaussian structure is a consequence of the squeezing that occurs both initially as well as over time. The final fit to the WCM data is done on the average of all bunches, since a bunch by bunch determination cannot be performed for the z vertex distribution. The parameters of these three distributions together is used to numerically integrate the formula for luminosity, generating the z vertex distribution. An example of a bunch averaged profile is shown in figure 2.14.

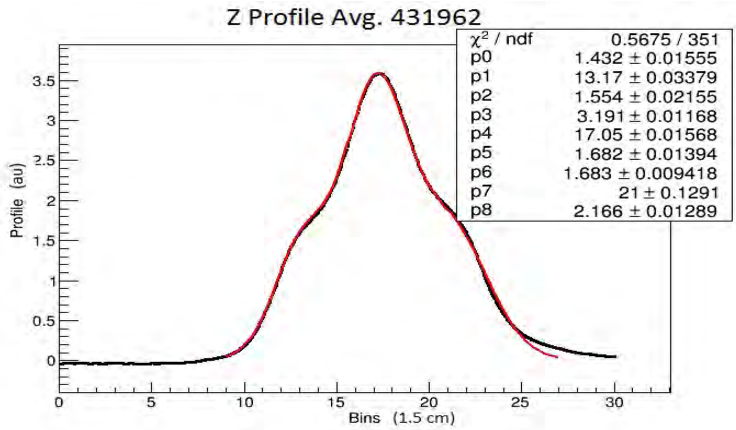
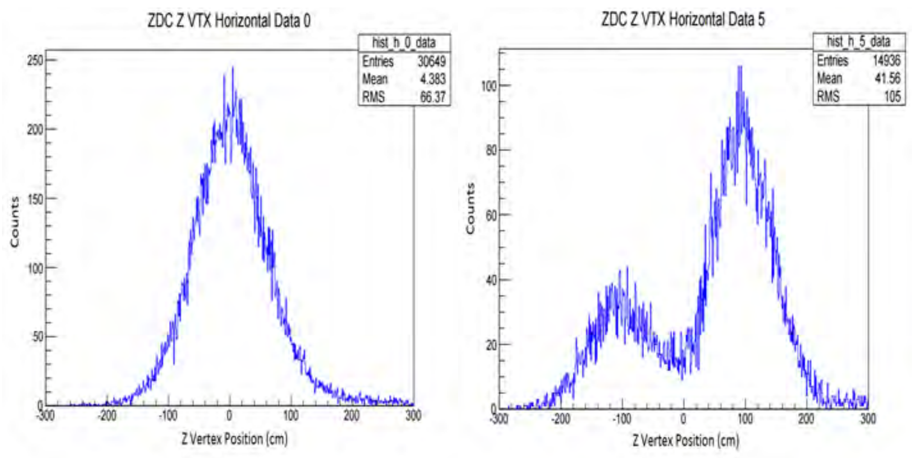



FIGURE 2.14: True Bunch Profile (blue) and Three Gaussian Fit (red). The fit parameters represent the fit to the three Gaussians that sum to give the bunch shape. p0-p2 are the fit parameters left Gaussian, p3-p5 correspond to the center Gaussian, and p6-p8 are the right Gaussian. The three parameters for each Gauassian are the normalization, mean, and width of the Gaussian respectively.

The parameter $\beta^*$ is the value of the beta function at the interaction point and is equivalent to the value of the $\beta$ function there. An infinite value of $\beta^*$ corresponds to a perfectly cylindrical bunch, and as the value decreases the bunch shape tends to an hourglass shaped cross section in the x-z and y-z planes. A non infinite value of $\beta*$ has the effect of increasing the luminosity closer to the IP, while simultaneously reducing the luminosity further from the IP. This is a consequence of the values of $\sigma_{x(y)}$ increasing dramatically further from the interaction point. While there is a distinct $\beta^*$ in both x and y, they are considered equivalent in the calculations.

Another necessary correction that is made to the simplistic luminosity integral is the presence of a small crossing angle $\theta$ between the beams. While it is possible to have a crossing angle in both the x-z and y-z planes, for PHENIX the crossing angle is largely in x-z plane and any effect in the y-z plane is taken to be negligible. The effect of this small angle is a reduction in the total luminosity. The crossing angle is again an effect that acts to impose a z dependence on the computation of $\mathcal{L}$. In this case, for the small crossing angle in the x-z plane, the coordinate system in which the luminosity is calculated is rotated through the angle $\theta$. However, since the angle is small and the value of x position relative to z position is also small, all terms of the form $xsin(\theta)$ are neglected, and $sin(\theta)$ is approximated as $\theta$ via the small angle approximation.

In order to quantify both the effect of $\beta^*$ and the crossing angle on $\sigma_{BBC}$, it was necessary to simulate bunch crossings. The simplest effect that the two parameters have on the Vernier Scan data is skewing of the z vertex distribution at large displacements in the transverse plane. The simplest case of a bunch crossing with infinite $\beta^*$, a crossing angle of zero, and Gaussian distributions in x, y, and z, the vertex distribution would be Gaussian for each Vernier Scan step. However, with a non infinite $\beta^*$, the bunches take on an hourglass shape, thereby leading to a Gaussian distribution at maximally overlapped steps, and a doubley peaked distribution at a greater transverse displacement. Secondly, the crossing angle causes a skew in the double peak structure of the z vertex distribution at minimal overlap; the angle forces one side of the interaction region to have a greater overlap of bunches, thereby causing more collisions on that side. This effect is easily observed by examining the ZDC z distribution of verticies data node in the Vernier Scan output.




(A) (B)

FIGURE 2.15: Run 431962 ZDC z Vertex Distribution (a) Max Overlap (b) Min Overlap. The double peak structure on the left is a result of the structure in z, revealing the effect of a finite $\beta^*$ parameter and a nonzero crossing angle.

In order to determine the value of $\beta^*$ and the crossing angle, a numerical distribution of the luminous region is generated from the overlap integrals, such that their z vertex distribution can be matched to data. The ZDC z vertex distribution was chosen because some of the effects spill over beyond the $\pm 144$ cm where the BBC's are located, and the ZDC's efficiency is not z dependent. The numerical integration derives its z verticies directly from numerically integrating the full form of the luminosity equation, including the effects of $\beta^*$ and crossing angle, as well as the true z structure of the bunches obtained from finely binned WCM data, integrating over x, y, and z to generate a single distribution in t. The distribution is normalized, and a distribution in z is generated by summing over all the bins in t. The z bins of the distribution are smeared for ZDC resolution effects with a Gaussian where the position resolution is $\sigma_= 15$ cm.

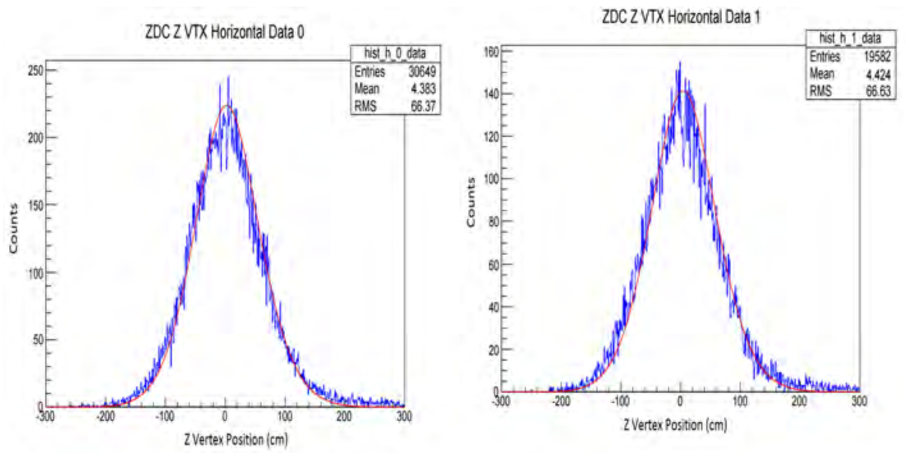
The generated histogram is then compared to the data histogram, and $\beta^*$ and the crossing angle are adjusted to converge to the best fit. The same program computes a scaled value of luminosity, which is first done with the physical values of the parameters found by comparison, then the computation of luminosity is repeated for the first order approximation of infinite $\beta^*$ and zero crossing angle. A normalization, $S$ is then computed by dividing the luminosity with physical values by the luminosity with first order values,

$$S = \frac{\mathcal{L}_{physical}}{\mathcal{L}_{first\ order}}$$

Given the possibility of systematic errors in the simulation, the computed value of the simulation luminosity is not considered be accurate enough to be used. Therefore, the eventual calculation of the delivered luminosity is modified to,

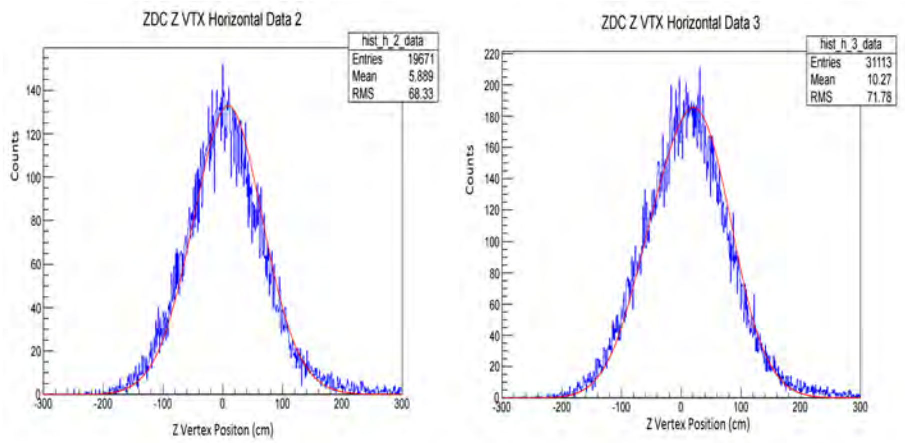
$$\mathcal{L}_{delivered} = S\mathcal{L}_{first\ order}$$

Example plots of the data and generated distribution are below. The steps are numbered from 0 to 11, and with 0 being the maximally overlapped configuration, 5 being minimally overlapped, 6 returning to max overlap and 11 minimally overlapped. All generated distributions have a $\beta^* = 90$ cm and $\theta = .06$ mrad.




(A) (B)

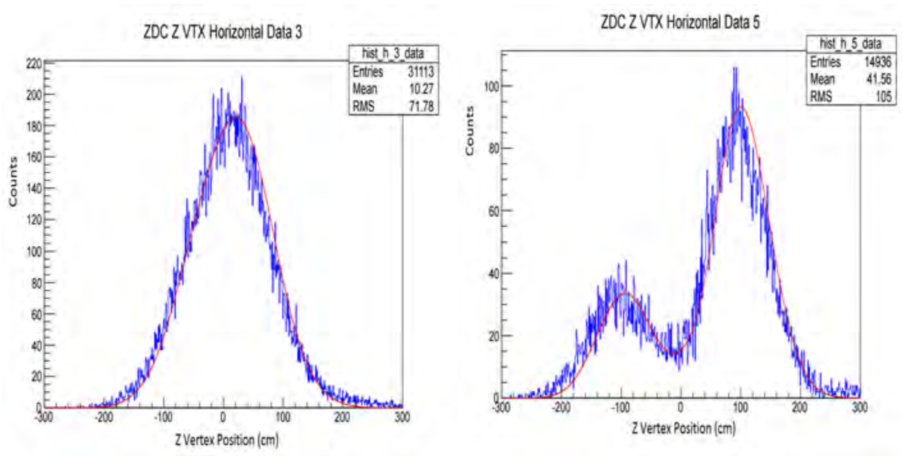
FIGURE 2.16: Run 431962 Data (blue) Generated (red) (a) Step 0 (b) Step 1




(A) (B)

FIGURE 2.17: Run 431962 Data (blue) Generated (red) (a) Step 2 (b) Step 3. This shows the correspondence between the generated distribution and the data for the z distribution of the verticies.

The evolution of the structure of the z vertex distribution is a consequence of the geometry of the interaction region. The finite $\beta^*$ causes the double peak structure for the minimally overlapped steps, whereas the crossing angle causes the asymmetry of the distribution. Ultimately, the complexity of the bunch structure and the coarseness of the generated distribution make it impossible to use the generated distribution as an absolute measurement of the two parameters. Therefore, the simulation is used to confirm with an uncertainty of approximately ten percent, that CAD's quoted values of $\beta^*$ and the crossing angle are correct.




(A) (B)

FIGURE 2.18: Run 431962 Data (blue) Generated (red) (a) Step 4 (b) Step 5

## 2.7 Systematic Errors

There were several sources of systematic error identified in the various steps of the Vernier Scan analysis. The final computed quantity, $\sigma_{BBC}$, can be broken down in terms of the parameters that comprise it.

$$\sigma_{BBC} = \frac{R_{max}}{\mathcal{L}_{BBC}} = \frac{R_{max}}{\epsilon_{trig} S \mathcal{L}_{delivered}} = R_{max} \frac{2\pi\sigma_x\sigma_y}{N_b N_y f_0 \epsilon_{trig} S}$$

In order to compute a final systematic error, the systematic error for each parameter is found separately, and errors are then propagated according to the formula:

$$\delta\sigma_{BBC} = \sigma_{BBC} \sqrt{\sum_{i=1}^{n} (\frac{\delta p}{p})^2}$$

Here, $p$ is any paramter and $\delta p$ is its associated systematic error. For the parameters that had identifiable sources of error, the systematic error was found first for the uncorrected value, then for each correction individually. The final systematic error on the parameter was the sum in quadrature of its uncorrected error and each correction's error.

The error estimation begins by looking at systematic errors of $R_{max}$. Since the uncorrected measurement of $R_{max}$ is simply BBC Trigger counts divided by CLOCK trigger or epoch time, it is assumed to be free of systematic error. Additionally, the systematic error resulting from the Luminosity fall off correction was neglected given the correction's 1% change and the 2% systematic on the WCM. To account for possible errors in the step positions, the Rate vs. Step plots were generated using first the CAD set steps, then the BPM measured steps. The difference in the value of $R_{max}$, about 1.5%, was used as additional systematic error. The multiple collisions effect did introduce systematic error into the determination of $R_{max}$ as discussed in the rate section. The multiple collisions error varies with displacement of the beam, and is averaged over all steps for each bunch crossing. The fractional error for the corrected rate was around 3%.

The systematic error on the intensity comes directly from the CAD quoted values of the systematic errors for the WCM and DCCT readings. CAD quotes the systematic error for the WCM at 2% and the DCCT at .2%. Given that $N_{b(y)}$ is normalized by the DCCT, the associated error, minus an approximated correlation, gives a final error of approximately .8%.

The error that resulted from $\sigma_x$ and $\sigma_y$ were computed as combinations of the rate error and the step position error. The rate error came by extracting values of $\sigma_{x(y)}$ from the uncorrected rate, then taking the fit parameter using the correct rate. The difference between the two was used as the error, and was approximately .5%. The error that came from uncertainty on step positions was computed similarly to the step position error on rate. The fit parameters were taken first using CAD set steps, then using BPM measured steps, and the difference was the possible error. This error was .7% for $\sigma_x$ and .5% for $\sigma_y$.

The systematic error in the parameter $\epsilon_{BBC}$ was taken to be the square root of the variance of all of the values of $\epsilon_{BBC}$ for all Vernier scans. The final value was for the error was .00439, which makes the relative error on the order of one percent, and specifically for 431962, the fractional error was 1.2%.

The correction of $\beta^*$ and the crossing angle had a high relative systematic error since the generated distributions and data plots were not matched to a high degree of precision. The spread around the best value that still appeared to give a tolerable z vertex distribution were used to compute correction factors. Once the factors were computed, the difference between them was used as an absolute error on the correction, and the fractional error was around 10%.

## 2.8 Results and Conclusion

Using only run 431962, the final value of BBC min bias detector cross section was,

$$\sigma_{BBC} = 30.0 \pm 1.8 \pm 3.4 \ mb$$

where the first error is statistical and the second is systematic. The overall statistical error is simply the standard deviation of the bunches, and the overall systematic error is the result of error propagation as defined in the previous section. The systematic error was approximately 11.3%, where the largest contribution comes from uncertainty in the $\beta^*$ correction.

The results were consistent with previous years' Vernier scans at PHENIX. However, those previous years at PHENIX have shown that the value for $\sigma_{BBC}$ can vary from run to run, more than the statistical variation within one run. This result will be presented to the collaboration and a plan will be developed for how to best account for this. Figure 2.19 below shows the values for $\sigma_{BBC}$ as a function of bunch number, and figure 2.20 is a histogram of the same values.

It is important to notice in figure 2.19 there is a group of bunches near bunch crossing 0 that are statistically higher than the remainder. These were included in the final average, because the discrepancy was thought to be a result of the fact that $\beta^*$ and crossing angle corrections cannot be made on a bunch by bunch basis. Since the bunch near the beginning of the beam (approximately 0-7) are following the empty bunches (107-120) known as the abort gap, there is less of a space charge effect on the beginning bunches. This is a well understood effect, and is seen in other Vernier scans in past run years [5], and therefore it is not a concern.

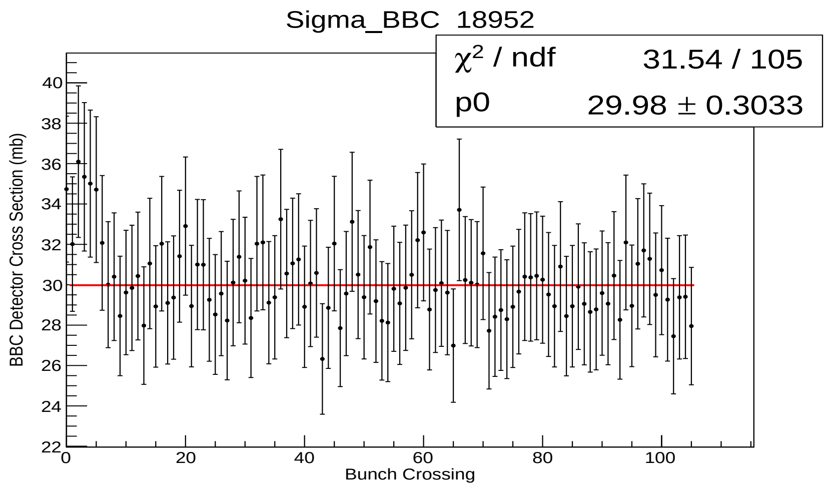



FIGURE 2.19: The collection of $\sigma_{BBC}$ values for 431962 with associated error. The fit was a $\chi^2$ fit to a constant function to all the data. The resulting value is parameter p0.

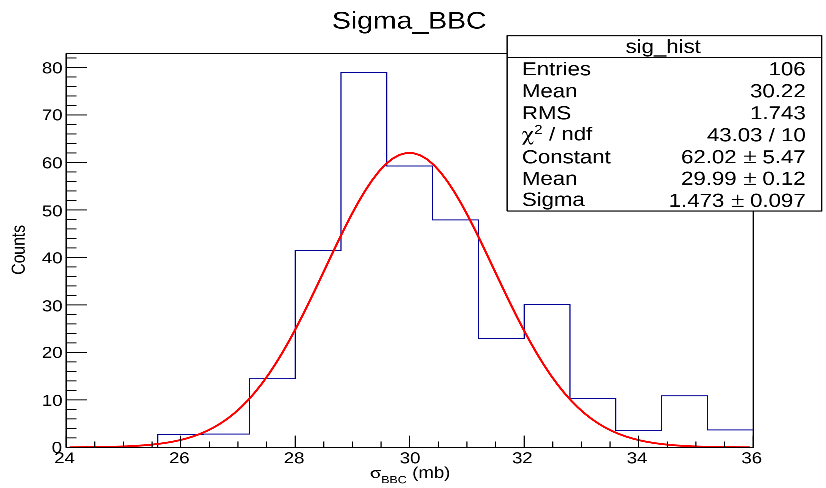



FIGURE 2.20: The histogram of all the $\sigma_{BBC}$ values for run 431962. The mean for the Gaussian fit to the histogram is consistent with linear fit to the data. However, given the $\frac{\chi^2}{NDF} = 4$, there is not a strong correspondence between the data and this Gaussian fit.

## Appendix A

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