The purpose of this class is to introduce students to two important pillars of mathematical physics, namely linear algebra and partial differential equations (PDEs), and discuss their applications to physical problems in mechanics, E&M, and quantum mechanics at the advanced undergraduate level. This class will provide a physics-based alternative to the more traditional PDE and linear algebra classes offered by the Mathematics department, and serve as a capstone math class for the physics and astrophysics majors as they begin junior-level physics classes. It will build on the foundational concepts of complex algebra, vector analysis, and ordinary differential equations. Numerical calculations based on Matlab, as learned in Phys 290, will be employed wherever purely analytical methods fail to provide simple closed-form expressions.

Bear in mind that since this class is not cross-listed with Math, it will not meet a Math minor course requirement. It may, however, be substituted for the Math 312 and 321 requirements under the BS program in Physics and Astrophysics. A petition will be necessary to perform this substitution until it is approved in the UNM catalog as officially meeting these math requirements for the Physics and Astrophysics majors.

Prerequisites – Math 311, Math 316, and Phys 290. The published (official) list of pre-requisites is incorrect, so please ask the instructor (email: sprasad@unm.edu) for a pre-requisites waiver, should you experience any difficulty in registering for this class.

Problems Class – Listed officially as Phyc 451.054 (CRN: 41498; Th 11-11:50 am, Rm 5), this is a **required** adjunct to the main lecture class. Please make sure you are enrolled in **both** Phyc 300.001 and 451.054 classes if you plan to attend.

Required Text – Mathematical Methods in the Physical Sciences by Mary L. Boas, 3rd ed., with more numerical approaches and examples to tie in with Phys 290

List of Topics (with applications from physics and engineering; 1 wk = 3 50-min lectures + 1 50-min problem session) -

- 1. Complex numbers (1 wk)
 - a. Complex plane, complex algebra
 - b. Trigonometric and exponential functions
- 2. Linear algebra (5 wks)
 - a. Vectors and matrices
 - b. Determinants, Cramer's rule
 - c. Matrix operations
 - d. Linear operators, linear independence, Gram-Schmidt orthogonalization
 - e. Eigenvalues, eigenvectors, diagonalization, similarity transformation
 - f. Unitary, normal, Hermitian, positive definite matrices
 - g. General vector spaces

- 3. Review of vector analysis (1 wk)
 - a. Quick review of vector algebra
 - b. Fields and vector differentiation grad, div, curl interpretation and applications
 - c. Gauss/Stokes theorems
- 4. Fourier series and transforms (3 wks)
 - a. Periodic functions and Fourier coefficients
 - b. Exponential, sine, and cosine series and transforms
 - c. Partial sums and Gibbs phenomenon
 - d. Discrete and Fast Fourier transform
 - e. Relation to Laplace transform
- 5. PDEs and special functions (5 wks)
 - a. First order quasilinear PDEs and method of characteristics
 - b. Second-order linear PDEs -- Laplace, heat, wave, and Schrodinger equations
 - c. Solution via separation of variables
 - d. Frobenius series solution, Legendre polynomials, spherical harmonics, Bessel functions