

Physics 480/581

Problem Session No. 8

Monday, 22 October, 2018

1. For the 2-dimensional metric,

$$\mathbf{g} = 2h^2(u, v) \underset{s}{du \otimes dv} = h^2(u, v) (du \otimes dv + dv \otimes du) ,$$

use the null basis set for 1-forms, $\varpi^{\hat{u}} \equiv h du$ and $\varpi^{\hat{v}} \equiv h dv$, so that the metric coefficients are constant. Then find the only non-trivial connection 1-form, $\Gamma_{\hat{u}\hat{v}}$. Using this connection 1-form, please now write out both the two geodesic equations for the tangent vector to an arbitrary but geodesic **timelike** path, and also the only relevant, non-zero curvature 2-form, $\mathcal{Q}_{\hat{u}\hat{v}}$, along with the curvature tensor component, R_{uvuv} .

Notes: The “guess” method of finding the connection 1-forms will not work; however, as there are only 2 components to one connection 1-form the coefficients may be determined fairly easily. Also be careful raising and lowering indices since the metric is not diagonal.

2. A brave galactic explorer is falling radially inward toward a black hole. She uses 4 basis vectors for her own reference frame, which we will call $\{\tilde{v}_{(\alpha)} | \alpha = \hat{r}, \hat{\theta}, \hat{\varphi}, \hat{t}\}$, which should be normalized to be orthonormal. Since her frame sees herself as at rest, it is reasonable to choose $\tilde{v}_{(\hat{t})} = \tilde{u}$, which causes her own measure of her own velocity as having only a non-zero component in the time position. Then, she should use spherical symmetry to make her angular basis vectors the same as the standard ones, which leaves one to have to determine only her radial frame vector. Having done this, determine the matrix which converts from our usual orthonormal tetrad to hers, determining her $\tilde{v}_{\hat{r}}$ from the normalization conditions. What is her 3-velocity as measured by the observer (more or less at infinity) who uses $\{r, \theta, \varphi, t\}$ as his coordinates? Do you recognize the matrix?