

Physics 480/581

Problem Session No. 7

Monday, 15 October, 2018

1. The Levi-Civita connection can be determined via derivatives of the metric (only) when using a coordinate basis for 1-forms:

$$\mathbf{g} = g_{\alpha\beta} dx^\alpha \otimes dx^\beta \iff \Gamma_{\mu\nu}^\lambda = \frac{1}{2} \{-g_{\nu\lambda,\mu} + g_{\mu\nu,\lambda} + g_{\mu\lambda,\nu}\} \omega^\lambda .$$

For the usual weak-field approach to general relativity, please calculate these connections, using $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ where all the components of $h_{\mu\nu}$ are assumed very small, so that products of them are to be neglected. Then, work out the form for the Riemann tensor in this case.

2. Write down the matrix for the transformation, in the Schwarzschild manifold, i.e., spherically symmetric and static, between a coordinate basis for 1-forms and the corresponding orthonormal basis. Using this matrix, show that the metric tensor components transform as we know they should between the two basis sets.