

## Physics 480/581

Problem Session No. 2

Monday, 10 September, 2018

1. Let  $\underline{B} = B_x dx + B_y dy + B_z dz + 0 dt$  be a 1-form representation of the 3-vector that describes the magnetic field, where the components depend on all of the usual Cartesian coordinates in spacetime, i.e.,  $\{x, y, z, t\}$ . Create the 2-form  $d\underline{B}$ , which then lives in the vector space  $\Lambda^2$ , and determine the components relative to the 6 basis vectors for this vector space. As an important aside what are the 6 basis 2-forms for  $\Lambda^2$ ?

2.  $T^\mu{}_\lambda$  are the components of a tensor of type  $[1,1]$ , as, perhaps, can be seen from the location of the indices, and is currently presented, as such a tensor, relative to the basis of that vector space, as

$$T = T^\mu{}_\lambda dx^\lambda \otimes \frac{\partial}{x^\mu} .$$

We want to change coordinates from the  $\{x^\mu | \mu = 1..4\}$  to a new set of coordinates,  $q^\alpha$ , which are (invertible) functions of the  $x^\mu$ . How do the components of  $T$  change?

3. An observer at rest notes a passing inertial reference frame moving with 3-velocity  $(3/5)\hat{x}$ . The passing frame is watching a moving ship and measuring its 3-velocity as  $4/5\hat{y}$ . What is our original observer's measurement of the 3-velocity of the moving ship? Use the transformation equations for the parts parallel to and perpendicular to the 3-velocity between frames.
4. Relevant to (3), what are the 4-velocities of the moving ship, as measured by the passing frame and by the original observer? Determine the boost matrix from the passing frame's measurements to the rest frame's measurements, and show that it relates them as it should.
5. The barycentric reference frame is the generalization, to special relativity, of the notion of the center of mass frame for more than 1 particle. For a system of particles it is the reference frame in which the total 3-momentum is measured to be zero! A proton with velocity  $v$  is headed toward another proton, which is at rest in a target. How fast is the barycenter moving? What is the invariant mass of the system?