Physics 480/581

Problem Session No. 12 Monday, 19 November, 2018

1. Suppose two equal mass, non-rotating black holes collide, and produce a black hole of some new total mass, also not rotating. Use the theorem that the area of the horizon of a black hole may never decrease to determine the maximum amount of the original masses, m each, may be radiated away.

2. A massive particle is dropped from rest at a particular value of $r \equiv r_0$, AND in the equatorial plane. Find equations for the values of the constants e and ℓ in terms of $dt/d\tau$, which would be the only initially non-zero component of the 4-velocity, and show that

$$e = \sqrt{1 - 2m/r_0}\ , \quad \ell = -\frac{2ma}{r_0 e}\ .$$

3. Defining the angular velocity of an object, as measured from infinity, as $\Omega \equiv d\varphi/dt$, show that if $\ell = 0$ and we are in the equatorial plane then $\Omega = \omega$, and that it therefore vanishes as $r \to \infty$, while it increases as r decreases. becoming just $a/(2mr_+)$ at the horizon. This is finite, and since the particle is observed—from infinity—spending infinite time approaching the horizon, it also implies that it will continue to spin around and around forever as it approaches.