Physics 480/581

Problem Session No. 11 Monday, 12 November, 2018

1. For the Kerr metric determine the area of the surface called the (outer) horizon, i.e., the surface with dt = 0 and

$$r = r_+ = m + \sqrt{m^2 - a^2}\ ,$$

which is where Δ vanishes, so that g_{rr} is infinite.

2. Since the scalar product of a 4-velocity with a Killing vector is a constant, then u_φ, and u_t are constants. However, $u^\mu = dx^\mu/d\tau$. How do these relate?

3. Given the standard orthonormal basis for 1-forms appropriate for the Kerr metric, find the corresponding reciprocal basis for tangent vectors.

$$\left.\begin{aligned} \underset{\approx}{\omega}^r &= \sqrt{\frac{\Sigma}{\Delta}} dr\ , \quad \underset{\approx}{\omega}^\theta = \sqrt{\Sigma}\, d\theta\ , \quad \underset{\approx}{\omega}^t = \sqrt{\frac{\Sigma\Delta}{A}}\, dt\ , \\ \underset{\approx}{\omega}^\varphi &= \sqrt{\frac{A}{\Sigma}} \sin\theta\, d\varphi - \frac{2mar\sin\theta}{\sqrt{\Sigma A}}\, dt = \sqrt{\frac{A}{\Sigma}} \sin\theta\, (d\varphi - \omega\, dt)\ , \end{aligned}\right\} \quad \underset{\approx}{\omega}^{\hat\mu} \equiv Y^{\hat\mu}{}_\alpha\, dx^\alpha$$

$$\text{with} \begin{cases} \Sigma \equiv r^2 + (a\cos\theta)^2\ , \\ \Delta \equiv r^2 + a^2 - 2mr\ , \qquad \omega \equiv \dfrac{2mar}{A} = -\dfrac{g^{\varphi t}}{g^{tt}} = -\dfrac{g_{\varphi t}}{g_{\varphi\varphi}}\ , \\ A \equiv (r^2+a^2)^2 - a^2\Delta\, \sin^2\theta = (r^2+a^2)\Sigma + 2ma^2 r\sin^2\theta = \Delta\Sigma + 2mr(r^2+a^2)\,. \end{cases} \tag{2}$$