Physics 480/581

Problem Session No. 10 Monday, 5 November, 2018

1. First consider the Faraday tensor, and show that any eigenvector it may have must be a null vector. Then consider the special case where both the electric and the magnetic fields are non-zero and only in the $\hat{z}$-direction. For this case find the eigenvectors and their eigenvalues.

2. If $\widetilde{K}$ and $\widetilde{L}$ are Killing vectors for some metric, show that the vector which is their commutator, i.e., $\widetilde{Q} \equiv [\widetilde{K}, \widetilde{L}]$ also satisfies Killing's equations.

3. Given the Kerr metric, create an orthonormal basis for 1-forms appropriate for it, and then find the corresponding reciprocal basis for tangent vectors.

$$\mathbf{g} = \Sigma \left(\frac{dr^2}{\Delta} + d\theta^2 \right) + \frac{A}{\Sigma} \sin^2\theta \, d\varphi^2 - 2\frac{2mar}{\Sigma} \sin\theta \, d\varphi \, dt - \left(1 - \frac{2m}{\Sigma} \right) dt^2 \,,$$

$$\Sigma \equiv r^2 + (a\cos\theta)^2 \,, \quad \Delta \equiv r^2 + a^2 - 2mr \,, \quad A \equiv (r^2 + a^2)^2 - a^2 \Delta \sin^2\theta$$