Senior Laboratory
PHYC 493L, Spring 2023

Classes: TR, 8 am – 11 am

Location: PAIS 1417

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Error in Measurement

Research Group	Reported value
Yale	15
Waterloo	15
UNM	12
UCSB	15

Error in Measurement

Research Group	Reported value
Yale	15 ± 7
Waterloo	15 ± 8
UNM	12 ± 2
UCSB	15 ± 4

Error in Measurement

Research Group	Reported value
Yale	15 ± 7 g/cm^3
Waterloo	15 ± 8 g/cm^3
UNM	12 ± 2 g/cm^3
UCSB	15 ± 4 g/cm^3

Density of gold = 19.3 g/cm^3
Density of lead = 11.4 g/cm^3

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Conclusion: It is very important to understand the reported errors on the UNM measurement.

Uncertainty is Necessary

- In research, a measurement without the uncertainty quoted is <u>wrong</u>.

- **"Error" does not mean mistake**

- Important related ideas:
  - Error and Uncertainty
  - Accuracy vs. Precision
  - Statistical vs. Systematic Uncertainty
  - Significant Figures
  - Resolution
  - Uncertainty in fitted data: errors on parameters vs. goodness of fit
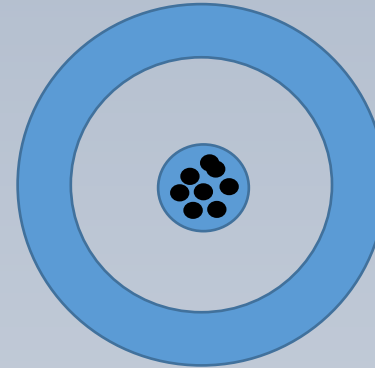
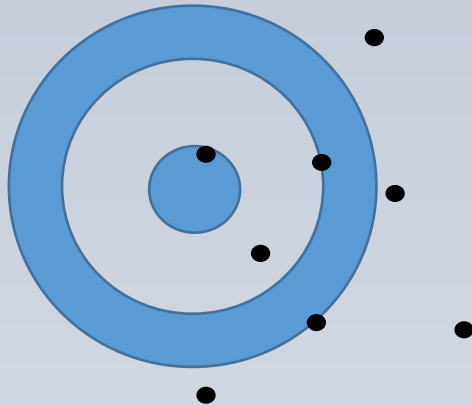# A word on Precision and Accuracy
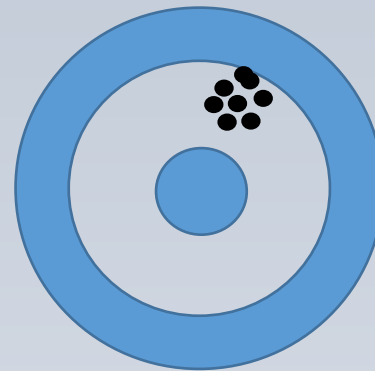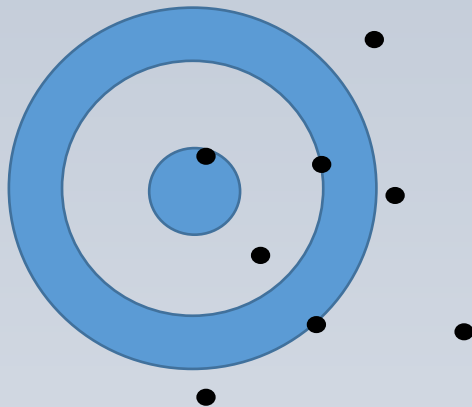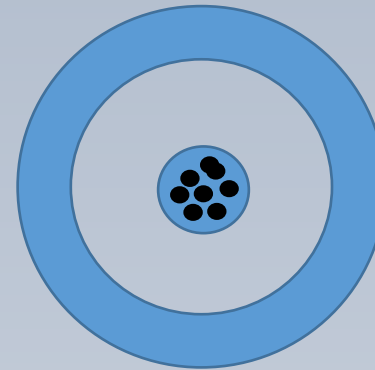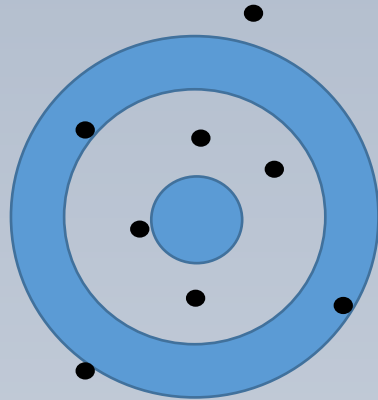
A word on Precision and Accuracy
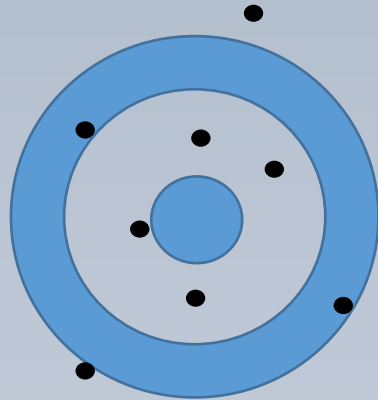
Large statistical uncertainty
Small systematic uncertainty

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Small statistical uncertainty
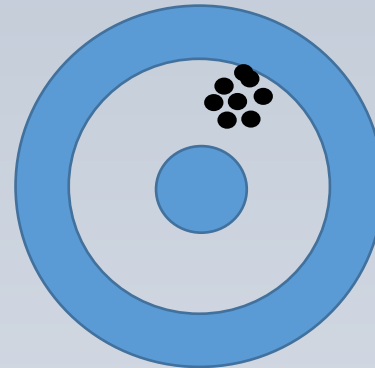Large systematic uncertainty

A word on Precision and Accuracy
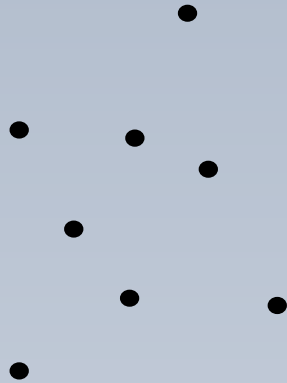
Large statistical uncertainty
systematic uncertainty = ??

Small statistical uncertainty
systematic uncertainty = ??

Large statistical uncertainty
systematic uncertainty = ??

Small statistical uncertainty
systematic uncertainty = ??

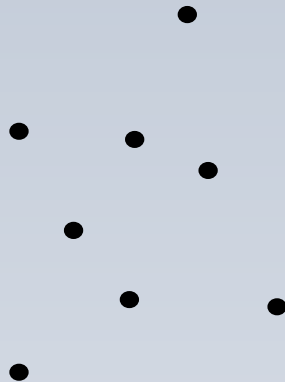It is always necessary to evaluate experimental/systematic sources of error/uncertainty, no matter how "good" your data.

Precision scientists often use "blinds" to prevent researchers from biasing data while it is being taken.

"We have learned a lot from experience about how to handle some of the ways we fool ourselves. One example: Millikan measured the charge on an electron by an experiment with falling oil drops, and got an answer which we now know not to be quite right. It's a little bit off because he had the incorrect value for the viscosity of air. It's interesting to look at the history of measurements of the charge of an electron, after Millikan. If you plot them as a function of time, you find that one is a little bit bigger than Millikan's, and the next one's a little bit bigger than that, and the next one's a little bit bigger than that, until finally they settle down to a number which is higher.

"Why didn't they discover the new number was higher right away? It's a thing that scientists are ashamed of—this history—because it's apparent that people did things like this: When they got a number that was too high above Millikan's, they thought something must be wrong—and they would look for and find a reason why something might be wrong. When they got a number close to Millikan's value they didn't look so hard. And so they eliminated the numbers that were too far off, and did other things like that ..."

--Richard Feynman, 1974

Significant Figures

Significant Figures

- The least significant digit tells you to what **precision** you've **measured** that value.

Examples:

- 200.006 meters
  - →How many sig. figs.?
  - →What is the measurement precision? (order of magnitude)

Significant Figures

- The least significant digit tells you to what **precision** you've **measured** that value.

Examples: (How many sig figs? LSD?)

- 0.0032

- 32.00

- 320

Significant Figures

- The least significant digit tells you to what **precision** you've **measured** that value.

Examples: (How many sig figs? LSD?)

- 320 (ambiguous!)

Breaking ambiguity:

320.

$3.2e2$ or 3.2×10^2

320 +/- 10

→Why does the last case break ambiguity?

Significant Figures

- Adding or subtracting:
  - <u>Answer has same LSD as the least precise measurement</u>

  13.04 s + 10.2 s

  103.42 - 0.42

Significant Figures

- Multiplying or Dividing:
  - <u>Answer has same number of sig. figs. as the measurement with fewest sig. figs.</u>

  13.05*10.0

  1105/5.0

How to report uncertainties

$$3.0 \pm 0.7 \text{ cm} = 3.0(7) \text{ cm}$$

How to report uncertainties

- Either:

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$$3.0(7) \text{ cm}$$

Almost always rounded to one sig fig:

$$3.0052 \pm 0.0004 \text{ cm}$$

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- Last sig fig in answer should usually be same order of magnitude as uncertainty

$$3.0 \pm 0.0004 \text{ cm}$$

How to report uncertainties

$$3.0 \pm 0.7 \text{ cm}$$

Not

$$3.0 \pm \sqrt{0.5} \text{ cm}$$

Mathematics of Error Propagation

Ask yourself:

Should I be thinking about <u>absolute</u> uncertainty

or <u>fractional</u> uncertainty?

Addition of measurements

10.7(3) ft + 9.3(4) ft

Addition of measurements

10.7(3) ft + 9.3(4) ft

= 20.0(5) ft

*Error reported = $\sqrt{(error1)^2 + (error2)^2}$

*for uncertainties which are **independent** and **random**

Subtraction of measurements

10.7(3) ft - 9.3(4) ft

Subtraction of measurements

10.7(3) ft - 9.3(4) ft

= 1.4(5) ft

Error reported = $\sqrt{(error1)^2 + (error2)^2}$

Note: Subtraction of large and similarly valued measurements can lead to a <u>big</u> increase in fractional uncertainty

Multiplication or Division of measurements

$1.4(1)$ kg $*$ $3.5(5)$ m/s^2

Multiplication or Division of measurements

$1.4(1)$ kg $* 3.5(5)$ m/s^2

$= 4.9(8)$ N

$$z = x * y$$

$$\frac{dz}{z} = \sqrt{\left(\frac{dx}{x}\right)^2 + \left(\frac{dy}{y}\right)^2}$$

(for uncertainties which are **independent** and **random**)

General formula for error propagation

$$y = f(x)$$

$$\delta y = \left|\frac{dy}{dx}\right| * \delta x$$

$$y = f(x_1, x_2, \ldots, x_N)$$

$$\delta y = \sqrt{\left(\left|\frac{\partial y}{\partial x_1}\right| * \delta x_1\right)^2 + \cdots + \left(\left|\frac{\partial y}{\partial x_N}\right| * \delta x_N\right)^2}$$

(for uncertainties which are **independent** and **random**)

Reporting uncertainty in your lab reports

- When reporting uncertainties, tell the reader where they come from. These could be:

  - The error bar on a fit. (A fit (to the expected function) gives a rate of 4.0(1) liters/s.)

  - The resolution of an instrument you used to measure. ("The analyzer had a resolution bandwidth of 100 kHz.")

  - The expected number noise on random events. ($\sqrt{N}$)

  - The standard deviation on repeated measurements. (We measure 100(9) microorganisms per sample.)

- Most often, one source of uncertainty dominates the uncertainty in your results. Learn to identify this!

Reporting uncertainty in your lab reports

- When reporting uncertainties, tell the reader where they come from.

- Most often, one source of uncertainty dominates the uncertainty in your results. Learn to identify this!

- Do not try to combine statistical deviation and uncertainty of in measuring devices. Report these separately!

Mean and standard deviation

Mean:

$$\mu = \frac{x_1 + x_2 + \cdots + x_N}{N}$$

(Population) Standard Deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}}$$

Sample Standard Deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N-1}}$$

Standard error is different:

Mean:

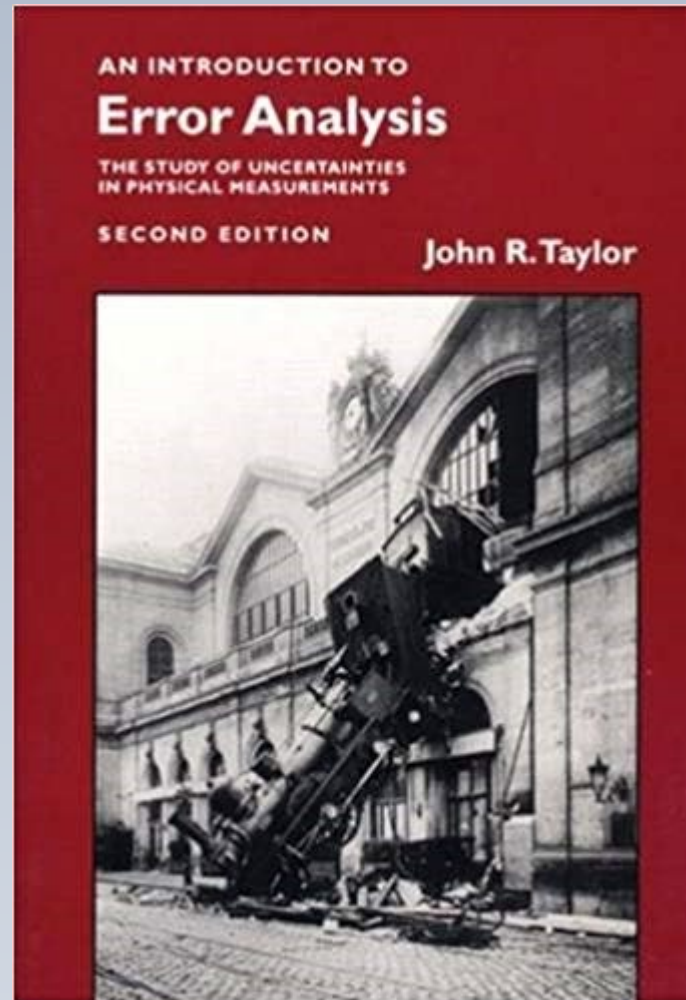$$\mu = \frac{x_1 + x_2 + \cdots + x_N}{N}$$

Standard Deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}}$$

Standard Error ("Standard Deviation of the Mean"):

$$\sigma_\mu = \frac{\sigma}{\sqrt{N}}$$

Error Analysis: an excellent reference

Homework #2

Let's go through it.

Coming Up:

- <u>Next Lecture</u>: fitting and uncertainty in MATLAB (next week)

- <u>Homework 2</u> (this one): send to me after class (or hand in next week)

- <u>Lab report 2</u>: report due 3/10 (Friday), late work will have points removed

- <u>Lab report 1 revisions</u>: due Thursday after spring break (3/23) for improved LR1 grade

- <u>Oral Presentations</u>: choose your topic by 3/9
    - Atomic hyperfine structure
    - Photon statistics for coherent vs. thermal (vs. quantum) light
    - Doppler free spectroscopy (and laser cooling of atoms)
    - Lasers: ECDLs, HeNe, ultrafast, and other types
    - Propose a related topic