# Lock-in Amplifier v1.0

1. Background

Lock-in amplification (also referred to as lock-in detection) is a method to recover weak but steady signals that may be buried deep in the noise background. This allows for the design of successful experiments even when the signal-to-noise ratio is much less than 1. The development of the lock-in amplifier is largely credited to Prof. Robert Dicke while at Princeton University.

The essential idea is as follows: An experiment is setup to measure the response of a system. A steady input signal V_{IN} is applied (eg. electrical current, laser light). In response to this input, the system produces an output signal V_{sig}. The output signal of the experiment will be accompanied by randomly fluctuating background noise that could originate from a variety of sources. The measured signal is: V_{sig} + Noise.

$$V_{IN} \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow V_{sig}+ \text{Noise}$$
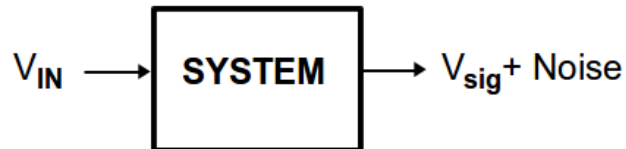
If V_{sig} is weak, Noise may be comparable to it, making the measurement unreliable. If Noise considerably exceeds V_{sig}, no useful data can be obtained.

To address this problem, V_{IN} is deliberately modulated at a known frequency ω. It is assumed that the system responds linearly to V_{IN} even though it changes from a DC ($\omega=0$) to an AC signal. The block diagram is modified as follows:

$$V_{IN}\sin(\omega t) \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow V_{sig}\sin(\omega t) + \text{Noise}$$
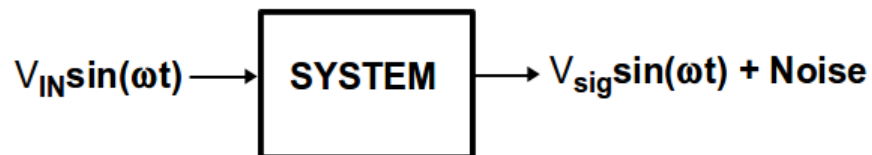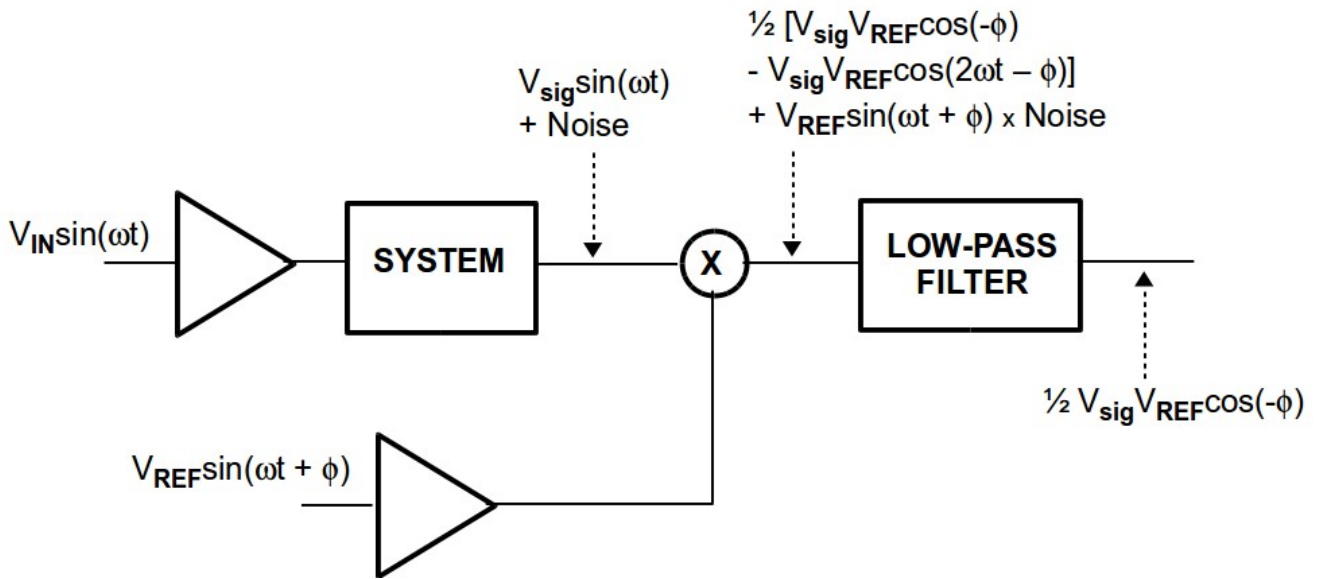
Notice that the noise is still present and unchanged in amplitude, but the output signal is now oscillating with the frequency of the input. At this point the signal could be sent through a band-pass filter at center frequency ω. This can sometimes work, but will fail if Noise exceeds V_{sig} in the passband, which is often the case.

The next step is to multiply the system output signal by a reference signal (V_{REF}) of the same frequency ω; we also allow for an arbitrary phase shift ϕ between V_{IN} and V_{REF}. The result of this multiplication is:

$$\tfrac{1}{2} [V_{sig}V_{REF} \cos(-\phi) - V_{sig}V_{REF} \cos(2\omega t - \phi)] + V_{REF} \sin(\omega t + \phi) \times \text{Noise} \qquad (1)$$

Eq. (1) shows that the new output signal emerges at three distinct frequencies: i) DC, ii) 2ω, and iii) ω. The unwanted noise is at ω, but there are two terms proportional to the desired signal (V_{sig}) at DC and 2ω. If the signal is sent through a low-pass filter to remove the ω and 2ω terms, the output is a constant, DC voltage that is directly proportional to V_{sig}. This is the key to lock-in

$$\frac{1}{2}\,[V_{sig}V_{REF}\cos(-\phi)$$
$$-\,V_{sig}V_{REF}\cos(2\omega t - \phi)]$$
$$+\,V_{REF}\sin(\omega t + \phi) \times \text{Noise}$$

$V_{sig}\sin(\omega t)$
$+$ Noise

$V_{IN}\sin(\omega t)$

SYSTEM

X

LOW-PASS FILTER

$$\frac{1}{2}\,V_{sig}V_{REF}\cos(-\phi)$$

$V_{REF}\sin(\omega t + \phi)$

amplification: a very weak signal can be spectrally separated from the background noise.

Observe that if the phase ϕ is an integral multiple of $\pi/2$, the desired signal vanishes. This means it is important to set the phase of the V_{REF} correctly to maximize the output.

A low-pass filter cannot be infinitely precise, i.e. it can't act as a mathematical δ-function at $\omega=0$. In PHYC 306L, passive and active low-pass filters were configured using poles to set the cutoff frequency and sharpness of the spectral response. The following Bode plot shows a low-pass filter with cutoff frequency (-3 dB) close to 1 Hz.
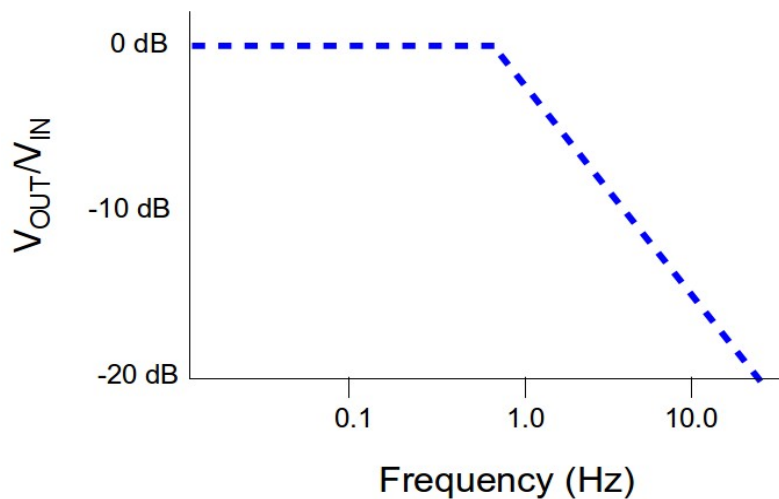
The closer the cutoff frequency gets to $\omega=0$ and the sharper the filter edge, the more accurate the lock-in measurement will be. The trade-off, however, is measurement time. As the cutoff frequency approaches $\omega=0$, more time will be needed to make a meaningful measurement. To illustrate, it is unacceptable to have a low-pass cutoff set at 0.1 Hz if the experiment is drifting on a timescale of 1 second. The experimenter must be aware of these issues when using a lock-in amplifier.

2. TeachSpin Lock-in Amplifier

The TeachSpin lock-in amplifier is a versatile instrument designed to introduce the principles and concepts to advanced physics students. Because it is a teaching tool, it looks considerably different than a lock-in amplifier found in a modern research lab. The fundamental operation, however, is identical.

In addition to the lock-in amplifier, you will need a dual-channel oscilloscope, a digital voltmeter, and some BNC cables/connectors. You will also need a notebook to record all information related to this experiment.

2.1 Characterization Experiments

Turn on the lock-in amplifier and scope. Start by connecting a BNC cable between the OUTPUT terminal of the Reference Oscillator to CH 1 of the oscilloscope. Trigger the scope on this channel and set the frequency to produce a 50--100 Hz sine wave (not critical). Set the amplitude to a convenient level for display; 1 V/div is good.

Use a BNC splitter (T- or F-connector) to send the OUTPUT signal in parallel to the INPUT of the Noise Generator. Use the special BNC cable with ground isolation, identified with clear plastic sheath that reveals the inner conductor (outer braid is missing). Set the Noise Amplitude to OFF and the Signal Attenuator to 10^{-2}. Put another BNC splitter on the OUTPUT of the Noise Generator and connect one port to Ch 2 of the oscilloscope. Confirm that a synchronous signal is present.

Connect the other port of the splitter to INPUT SIGNAL of the Lock-in/Amplitude Detector. To show the sensitivity of lock-in detection, it will be compared to direct detection, i.e. the RMS voltage of the input waveform will be measured first. Recall that the RMS voltage of an AC signal corresponds to the DC voltage that would produce equivalent power dissipation in a resistor. RMS is used because the average value of a sinusoidal waveform with no offset voltage is zero. Set the switch for AMPLITUDE DETECTOR. Connect OUTPUT to a DC voltmeter. Use DC coupling and set the GAIN for a signal < 0.5 V (not critical).

The next step is to increase the Noise Amplitude from the Noise Generator. Record the DC voltage as a function of this amplitude. You should see the desired signal level disappear as the contribution from noise overwhelms it. Switching over to lock-in detection will dramatically improve this situation.

Flip the switch to LOCK-IN DETECTOR and turn Noise Amplitude to OFF. Leave all other settings the same. Connect a BNC jumper cable from OUTPUT PHASE SHIFTER to the input terminal of the Phase Shifter. The output of the Phase Shifter is the INPUT REFERENCE for the lock-in. The OUTPUT of the lock-in is connected to the INPUT of the Low Pass Filter and its output can then be connected to the DMM. As with direct detection, it will display a DC voltage. Adjust the phase in the Phase Shifter to maximize this signal. You can amplify the DC signal with the gain knob to put it at a convenient level. This will be the noise-free, DC reference voltage. The goal is to maintain this signal as noise is introduced.

Repeat the systematic addition of noise as was done with direct detection. As the noise increases, you will likely have to adjust the the low-pass filter slope and time constant to maintain the desired signal. Filtering improves as the time constant increases, but realize that it may take 5 or more time constants for the measurement to stabilize. Can you detect a signal when noise is maximum? It is helpful to observe the low-pass output voltage on Ch 1 of the scope instead of the DMM to understand how it varies with time. Make an estimate of the signal stability in the presence of large noise levels and include in your report.

2.2 Demonstration Experiments

The next part of this lab is to use the same TeachSpin instrument to detect weak signals in a practical experiment. The central idea is to have an experiment that can be modulated and detected at a known frequency. This reference frequency of the modulator is used by the lock-in amplifier.

The student teams should propose a suitable experiment to the instructor. Here are some possibilities:

i) The output of a light-emitting diode (LED) can modulated by a sinusoidal or rectangular wave and measured with a photo-detector as a function of separation distance. This is very similar to an experiment done in 306L.

ii) The LED can be replaced with a laser and instead of modulating its output power, the laser beam is chopped (on-off) with a spoked wheel. This setup can maintain a detectable signal at dramatically larger separation distances compared to experiment 1.

iii) Instead of using light, consider audio/acoustic signals. A speaker source and microphone detector are available.

There is a preamplifier and band-pass filter available on the TeachSpin device that may help performance, but you are not required to use these features. Consult the instructor on their use. You need to do a careful, controlled experiment to characterize sensitivity and performance. Data must be recorded, analyzed, and presented in a technical report to tell a clear story.

3.0 LabView program

Implement a lock-in amplifier using LabView. If you took 306L, this should be familiar as it was one of the experiments. The primary difference here is that instead of using an Elvis board, a National Instruments DAQ card (USB-6008 or 6009) will be setup to process the analog signals. The DAQ instrument interface will be almost identical to the Elvis board configuration and makes use of DAQmx palette in LabView. You can use your own laptop or a lab computer, although the latter must remain at PandA. The performance of your LabView lock-in program should be compared to the TeachSpin device using the experimental procedure in Section 2.1 or Section 2.2.

4.0 Technical writeup

Each student on the team is responsible for producing a separate report. The writeup should follow the format of a formal technical document that you would see in a physics journal. There should be an abstract that concisely summarizes what you have done. For example: "A series of experiments have been performed to characterize the performance of two different lock-in amplifier configurations... It was found that...." Next there is an Introduction that orients the reader to the work with background material. There should be a section that clearly describes the experiment with diagrams and details. This is followed by sections for Results (graphs and/or tables are almost mandatory), Analysis/Discussion, and a short Conclusion. References are listed last. Use a template from a research journal of your choice (eg. APS, OSA). Search online or simply look in the hallways of our physics building for plenty of examples. There is no page requirement, but be sure to write clearly and concisely. Imagine writing for a reader who is technically capable, but does not understand what a lock-in amplifier does or how it works -- this would correspond to most graduate students in PandA. In your introduction, derive Eq. (1).