Poisson Statistics

1 Introduction

In this experiment you will study one of the most important statistical distribution in physics, the Poisson distribution. It describes the results of experiments where events occur at random, but at a definite average rate (for example radioactive decays). It is of paramount importance in all of atomic and subatomic physics, in particular.

2 Structure

You will use the same setup as the electron rest mass experiment. You must become familiar with (and investigate) γ-ray interactions in matter, the Sodium Iodide (NaI) detector including the Photomultiplier Tube (PMT), and the Multichannel Analyzer (MCA) Universal Computer Spectrometer (UCS). See the enclosed handout (in Ref. [2]). The MCA is a computer-controlled spectrometer (UCS30) that digitizes the pulses in the range (0-10)V and with risetimes from (0.5-30)μs.

Check the polarity of the high voltage for the PMT provided by the UCS, and do not exceed the maximum HV indicated on the NaI detector (If in doubt, ask before turning on the HV.) The PMT output needs to be connected to the MCA input. The raw anode output of the PMT should be a negative pulse, explain why. Observe the PMT output in an oscilloscope as you change the HV (gain of the PMT increases with increasing HV - make sure you understand why).

In the first part of the experiment, no gamma source will be needed as you will simply measure the background count rate in our NaI detector. You will use our multichannel analyzer (MCA) in multichannel scaling mode (MCS) instead of pulse height analysis mode (PHA). In MCS function, the MCA no longer acts as a pulse height selector, but as a multichannel scaler with each channel acting as an independent scaler. At the start of operation, the MCA counts the incident pulse signals (regardless of their amplitude) for a certain dwell time, and stores this number in the first channel. It then jumps to the next channel and counts for another dwell time period, after which it jumps to the next channel and so on. In MCS mode, therefore, the channels represent bins in time. Typical dwell times will be in the millisecond range.

Suggested starting values: High-voltage bias of 1000V for the NaI, minimum gain (both fine and coarse) for the preamp/amp /discriminator, and no radioactive source.

Setup the software with 256 channels (Note: this means the computer will effectively perform this simple counting experiment 256 times for you) and MCS mode of operation with 1 pass. Make sure the MCS is set to internal with the presets on. And now the important part: adjust the dwell time such that the average count rate per channel is around 1-2 counts. Save the resulting spectrum in an ASCII or text file in a folder with your last name(s) in the computer. Repeat this procedure for two other dwell times such that the average count rate per channel is around 5 counts and around 10 counts, respectively.

- Discuss the source of these background counts (where does this background come from?)

Repeat the experiment for a radioactive source, for example Cs^{137} adjusting the dwell time to obtain the same average count rates of around 1-2, 5 and 10 counts respectively.

3 Analysis

Plot your three resulting distributions with statistical error bars [1] for the two cases, (a) background counts in the detector and (b) when using a radioactive source. Use MATLAB. Notice the significant asymmetry of the distribution for the lowest average count rate (Poisson at work!). Calculate and standard deviation of the distribution in each case. How closely do your results follow the expected Poisson distribution, i.e. that the standard deviation is equal to the square the mean? Make a quantitative comparison. Also notice how your highest average count rate case is rather symmetric, i.e. already for an average count rate of around 10 the Poisson distribution is practically indistinguishable from a Gaussian.

Compare your three count rate distributions with the expected Poisson distributions graphically with statistical error bars. Calculate the chi square (χ^2) per degree of freedom. You can review the chi-square test in "Statistical Treatment of Data" under additional

[1] "The Poisson distribution and statistical uncertainties do not apply solely to experiment where counts are recorded in unit time intervals. In any experiment in which data are grouped in bins according to some criterion to form a histogram or frequency plot, the number of events n_i in each individual bin will obey Poisson statistics with a certain mean $\mu_i = n_i$ and fluctuate with statistical uncertainties" $\sigma^i_{Poisson}$ for each time bint (see Chapter 3 [3]).

resources or in Section 4.3 in Ref. [3]) where

$$\chi^2 = \sum_j \frac{[y_{exp}(j) - y_{theory}(j)]^2}{\sigma_j^2} \tag{1}$$

Discuss the goodness of the assumption; how well does the experiment approach the theory? For the highest average count rate case, repeat with a Gaussian distribution. Provide an explanation for this.

In general, Poisson (P) and Gaussian (G) distributions are not the same. Evaluate the Gaussian distribution at the same discrete x-values as the ones defined for your Poisson distribution. Next, normalize both distributions and plot the relative difference of the Poisson distribution from your data and the Gaussian distribution, i.e. (Poisson-Gaussian)/Poisson, for the case without a source, only with background counts for the three investigated average count rate per channel. This quantity is a measure of the asymmetry of the Poisson distribution around the mean. Compare this difference using your results with the theoretical prediction for these difference from Ref. [4].

$$\frac{(G - P)}{P} \simeq \frac{\delta - \delta^3/3\mu}{2\mu} \tag{2}$$

with $\delta = n - \mu$ where μ and $n = 0, 1, 2, ...$ are the parameters of the Poisson distribution. Discuss your results.

4 References

[1] Melissinos and Napolitano, Chapter 10.

[2] Multichannel Analyzer manual can be found in:
http://www.spectrumtechniques.com/ucs30.htm

[3] Data Reduction and Error Analysis for Physical Sciences, 3rd ed. Philip R. Bevington, D. Keith Robinson.

[4] L. J. Curtis, Am. J. Phys. **43**, 1101 (1975).